

Ice Sheet System Model 2012

Code Validation Guide

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Chapter 1

EISMINT Ice Shelf

The tests described in this chapter follow the work of [Rommelaere \[1996\]](#).

1.1 Test 1: Mass conservation

1.1.1 Setup of the experiment

We use an idealized geometry of a square ice shelf of length 200 km. The velocity is imposed everywhere as $u_0 = 400$ m/yr and the initial thickness is 500 m.

The boundary conditions are free radiation on all sides except on the left boundary where a thickness wave is imposed:

$$H(t) = H_0 + \frac{H_0}{5} \sin\left(\frac{2\pi t}{T}\right) \quad (1.1)$$

We want to check whether the mass conservation scheme is able to propagate the signal along the shelf. The analytical solution is:

$$H(x, y, t) = H_0 + \frac{H_0}{5} \sin\left(\frac{2\pi}{T} \left(t - \frac{x}{u_0}\right)\right) \quad (1.2)$$

1.1.2 Results and comparisons

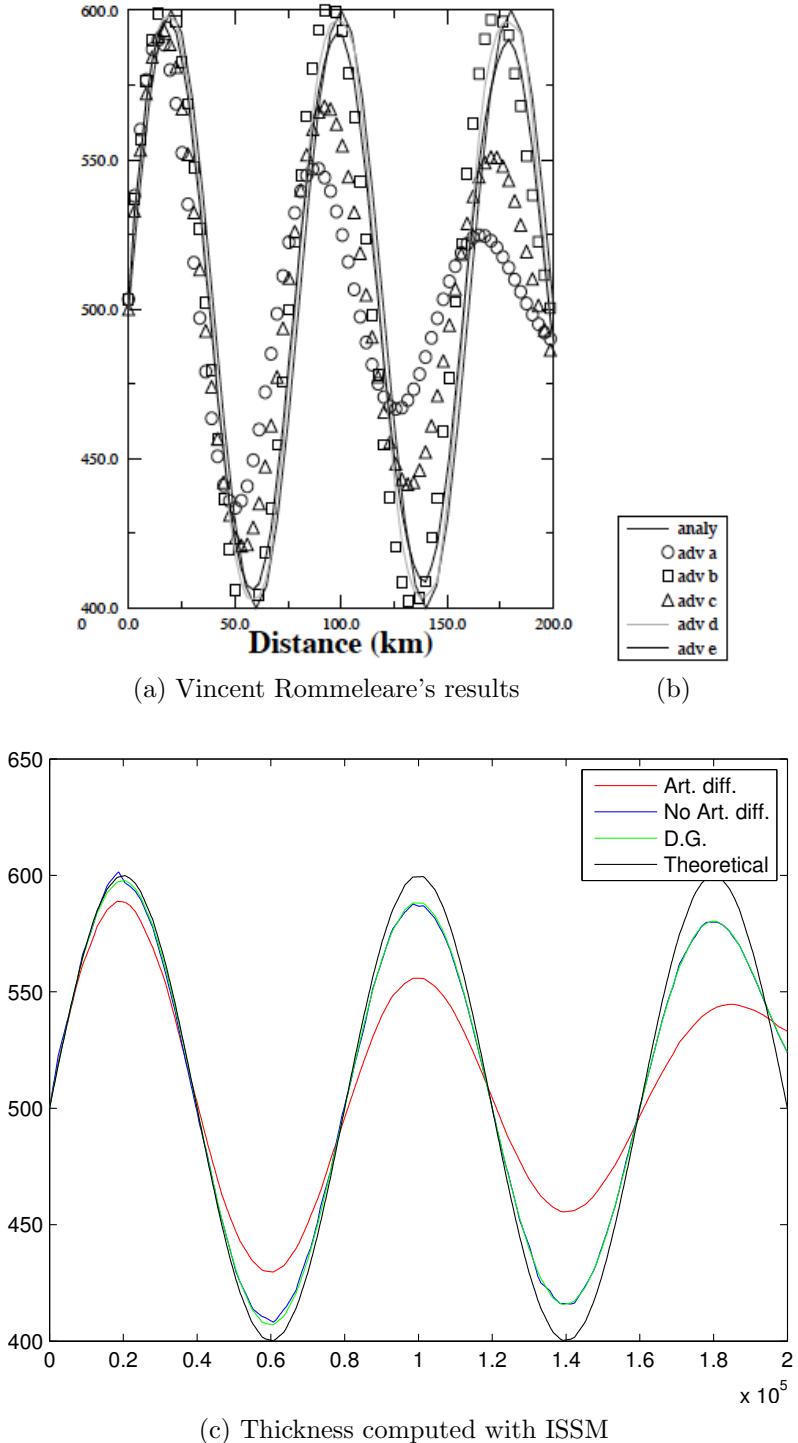


Figure 1.1: Comparision of the ISSM results with Vincent Rommeleare's

1.2 Test 2: Diagnostic 1

We use a square ice shelf with an imposed velocity everywhere except on the front where a dynamic boundary condition is imposed (water pressure).

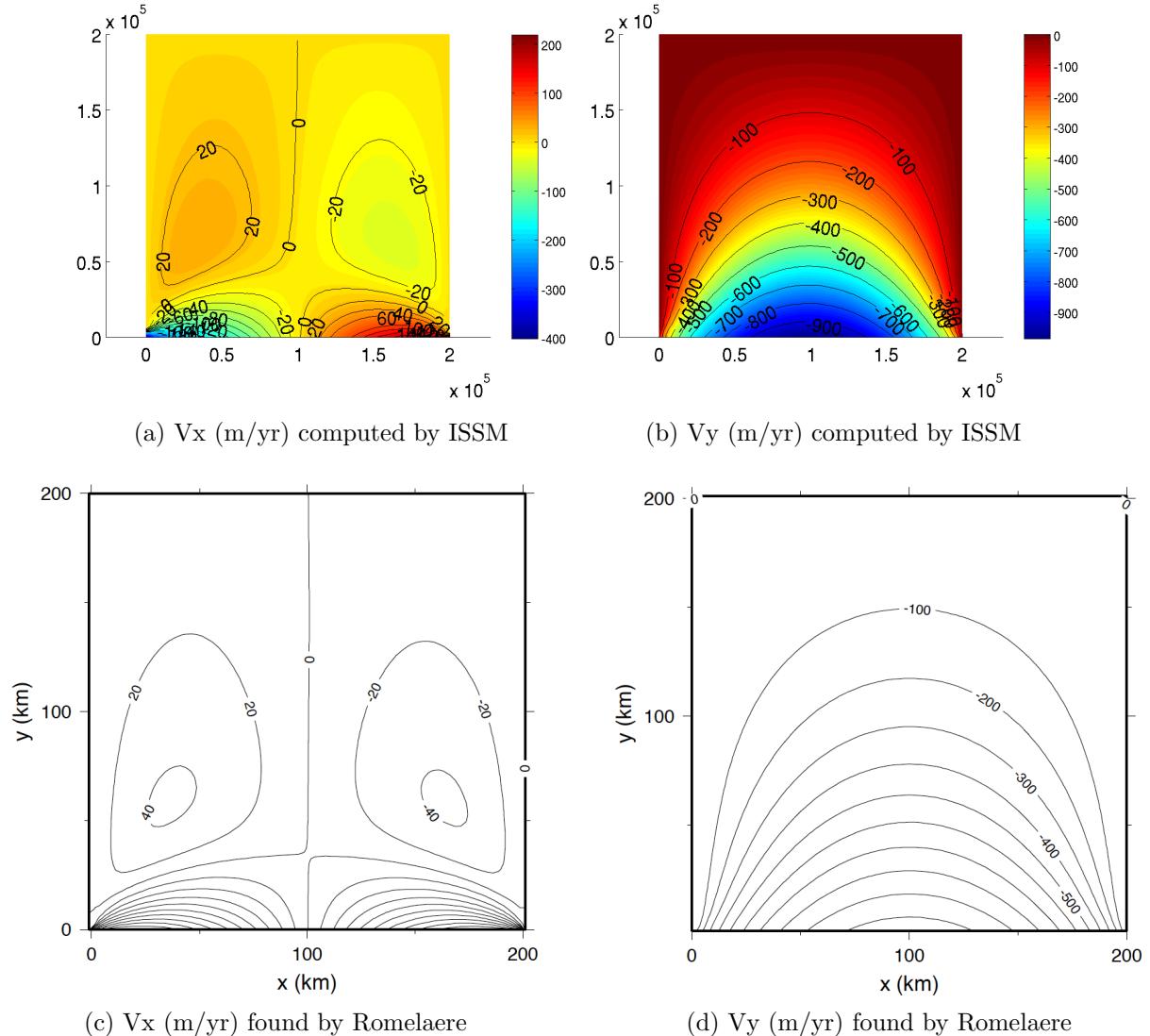


Figure 1.2: Comparision of the ISSM results with Vincent Rommeleare's

1.3 Test 3: Diagnostic 2

The geometry is exactly the same as the previous one except that the upper boundary condition is changed to account for an ice stream entering the ice shelf. The y-velocity becomes:

$$v(x, (y = L)) = 400 \left(\left[\frac{x - 100}{25} \right]^2 - 1 \right) Heav \left(1 - \left[\frac{x - 100}{25} \right]^2 \right) \quad (1.3)$$

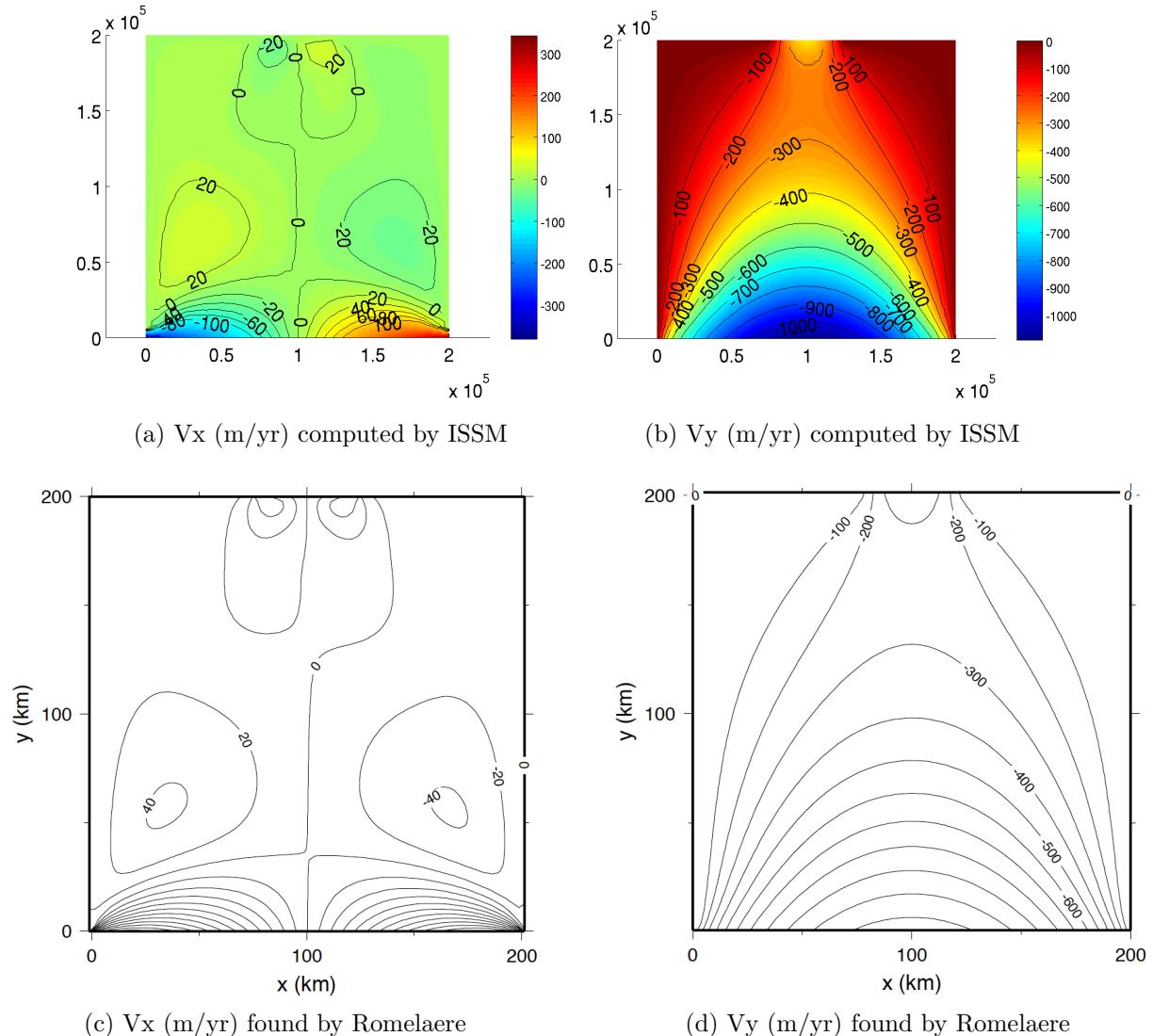


Figure 1.3: Comparision of the ISSM results with Vincent Rommeleare's

1.4 Test 4: Transient 2

This test starts from results of test 3. Using the same geometry as in the previous test, we let the model evolve for 5,000 years with an accumulation of 0.2 m/yr. The results after these 500 years are:

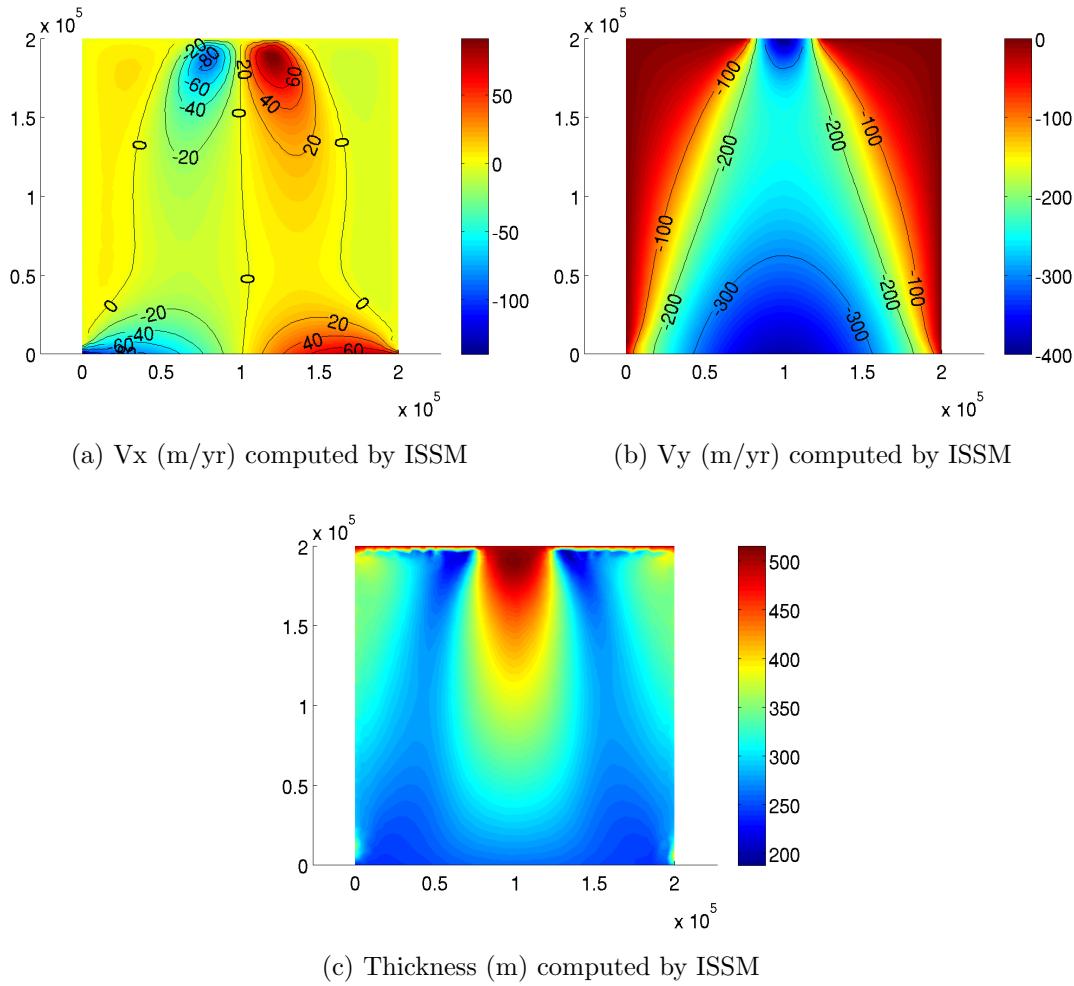


Figure 1.4: Ice shelf after 5000 years of evolution

Chapter 2

EISMINT Ice Sheet

The tests described in this chapters follow the EISMINT benchmark for ice sheets [[Huybrechts et al., 1996](#); [Payne et al., 2000](#)].

2.1 Test 1: Static case

2.1.1 Setup of the experiment

The EISMINT (European Ice Sheet Modeling INiTiative) intercomparison group proposes several tests to compare models to each other. One of them is a square ice-sheet for which there is not an exact, analytical solution for the steady-state ice sheet surface profile. To achieve a comparison between the modeled and the exact solution¹, we use a circular domain (with azimuthal symmetry) as shown in Fig.2.1a. With a couple of assumptions, the surface of the steady-state glacier follows the equation:

$$s(r) = \left(\frac{5\dot{M}_s}{2(\rho g)^3 A} \right)^{1/8} \left\{ 4 \left[\left(\frac{R}{2} \right)^{4/3} - \left(\frac{r}{2} \right)^{4/3} \right] \right\}^{3/8} \quad (2.1)$$

- $A = B^{-n}$ is the flow law parameter
- \dot{M}_s is the accumulation rate in meter ice equivalent, assumed to be constant

With $\dot{M}_s = 0.3 \text{ m.a}^{-1}$, $A = 10^{-16} \text{ Pa}^{-3}\text{s}^{-1}$ and $R = 750 \text{ km}$, the surface has the following shape:

¹This analytical solution was apparently derived by Nye and by Vialov. This references is in P. Huybrecht's paper on the EISMINT intercomparison test presented in September 1995 at Chamonix.

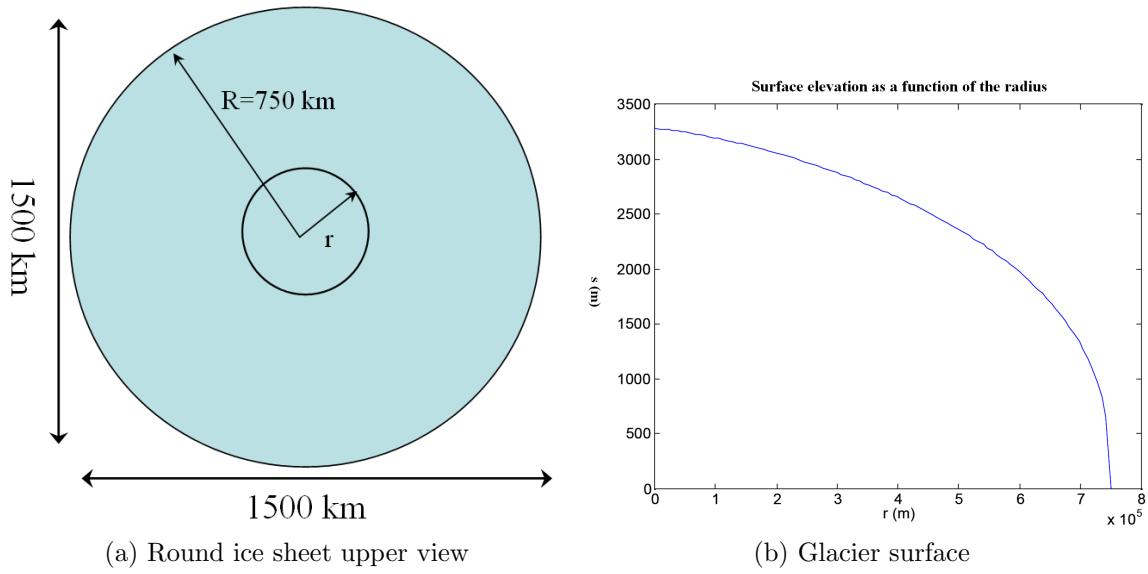


Figure 2.1: Round ice sheet geometry

The depth averaged radial velocity has an analytical expression:

$$\bar{u}(r) = \frac{r^2 \dot{M}_s}{2} \left(\frac{5 \dot{M}_s}{2(\rho g)^3 A} \right)^{-1/8} \left\{ 4 \left[\left(\frac{R}{2} \right)^{4/3} - \left(\frac{r}{2} \right)^{-4/3} \right] \right\}^{3/8} \quad (2.2)$$

2.1.2 Results with Hutter

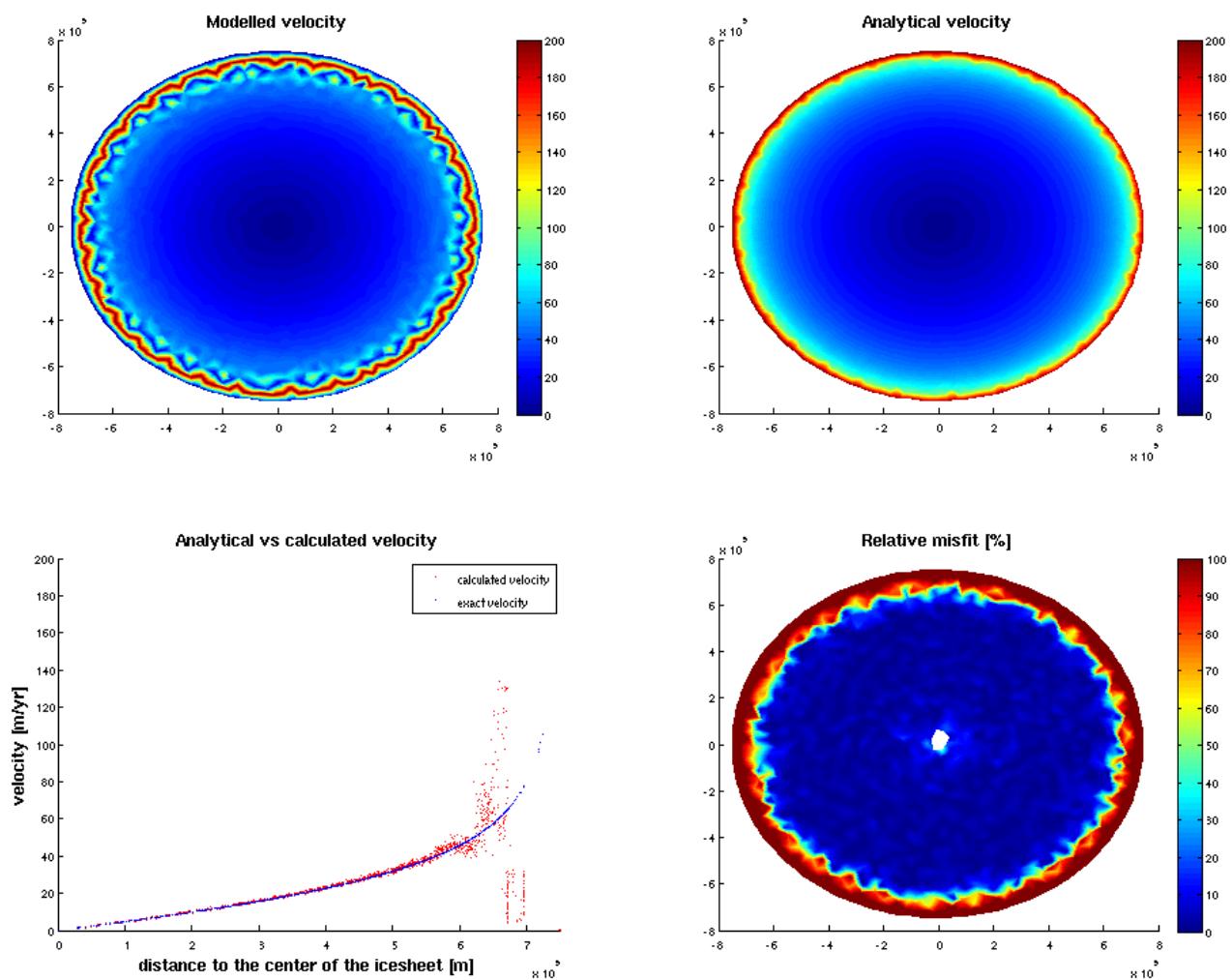


Figure 2.2: Comparison between analytical and modeled velocity

2.1.3 Results with Blatter/Patty

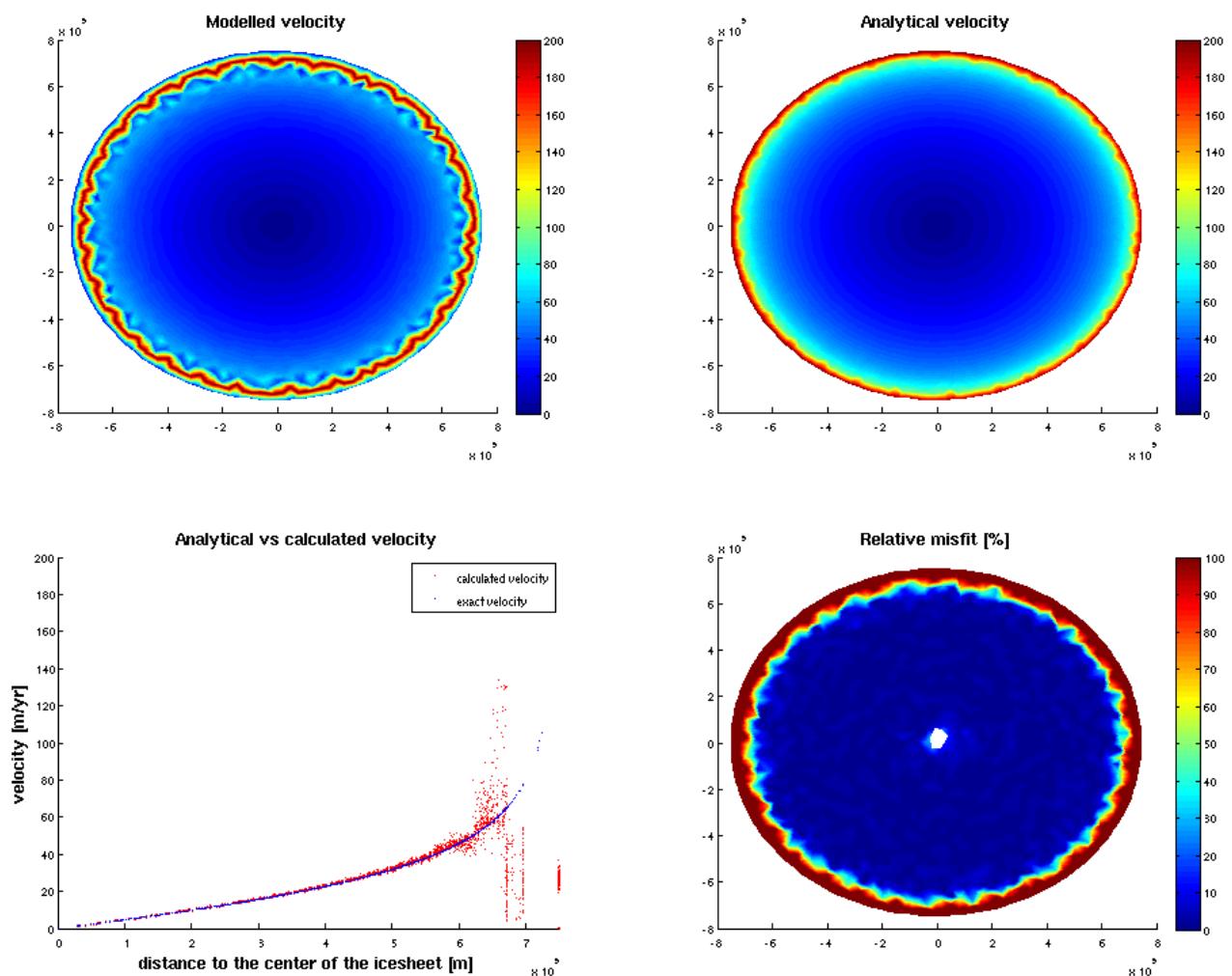


Figure 2.3: Comparison between analytical and modeled velocity

2.1.4 Results with full-Stokes

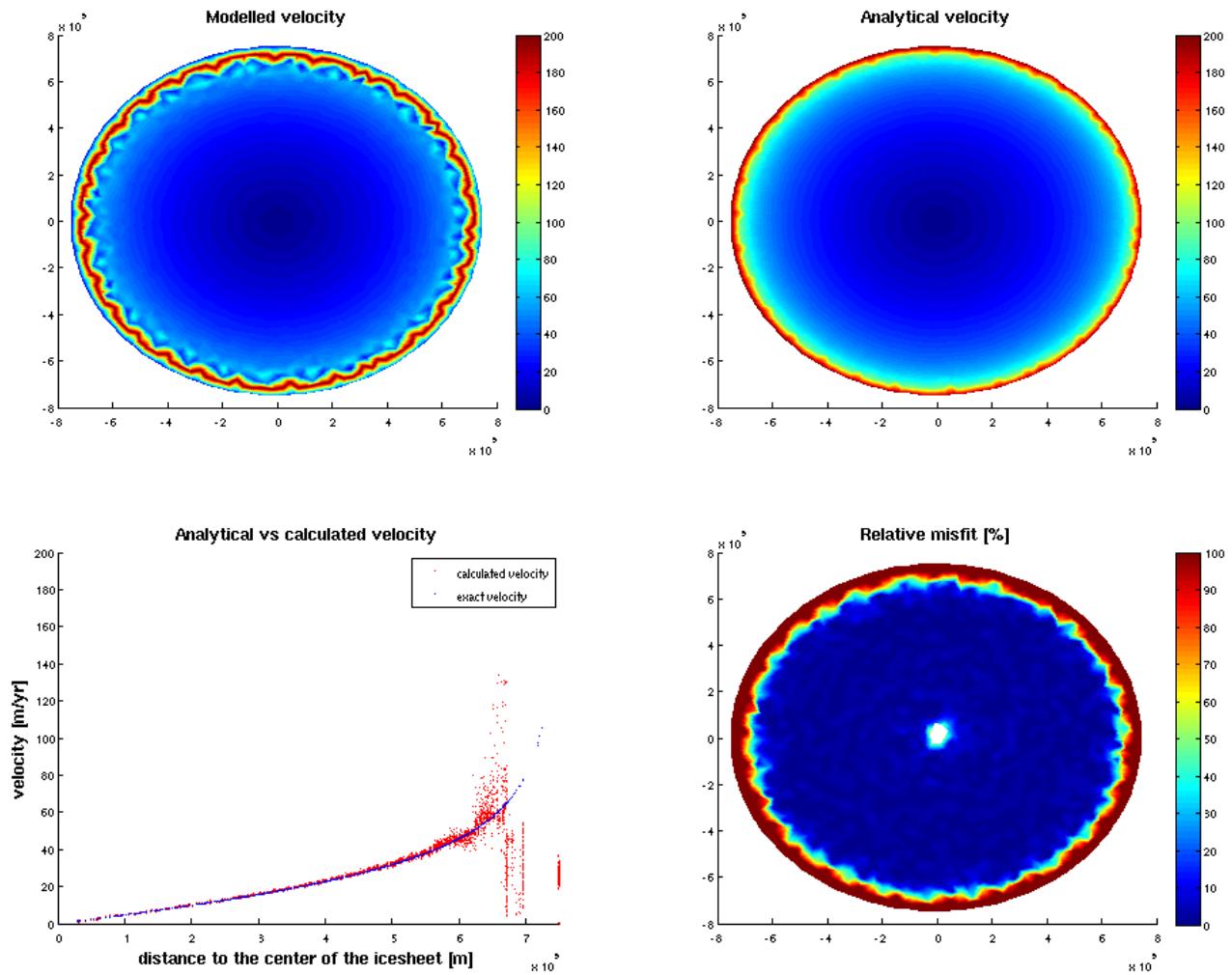


Figure 2.4: Comparison between analytical and modeled velocity

2.2 Test 2: Experiment A

2.2.1 Setup of the experiment

This test is close from the previous one, but this time it is time dependent (Experiment A in [Payne et al. \[2000\]](#)). We start from a square domain and no ice and we let the ice sheet evolves until it reaches a steady-state, which should take around 50s, 000 years. The parameters distribution has a round symmetry so we expect the results to have this same symmetry.

2.2.2 Results

Chapter 3

ISMIP-HOM

The tests described in this chapters follow the works of [Perichon \[2007\]](#) and [Pattyn et al. \[2008\]](#). For a comprehensie description of the experiment, please see [Pattyn and Payne \[2006\]](#).

3.1 Test A

3.1.1 Geometry

This is a 3d ice-sheet flow over a bumpy bed experiment. Periodic boundary conditions are applied. The geometry follows:

- surface $s(x, y) = -x \tan(0.5^\circ)$
- bed $b(x, y) = s - 1000 + 500 \sin\left(\frac{2\pi}{L}x\right) \sin\left(\frac{2\pi}{L}y\right)$
- $5 \text{ km} \leq L \leq 160 \text{ km}$

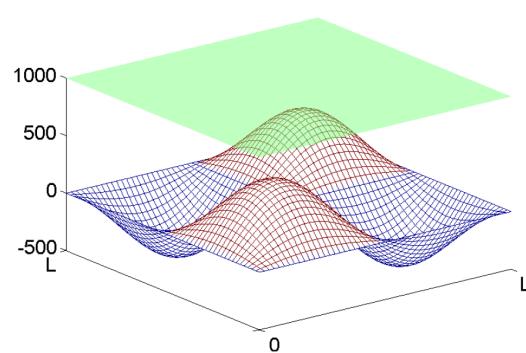
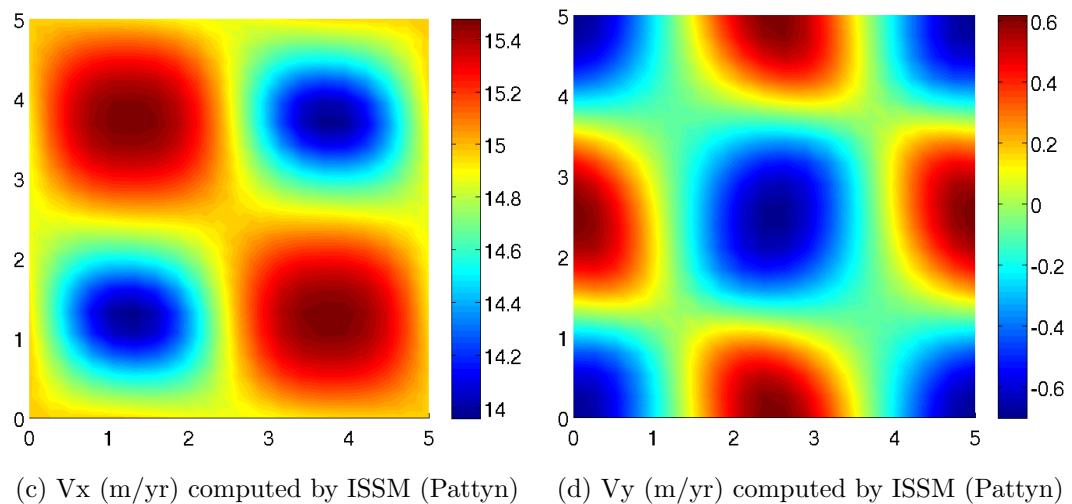
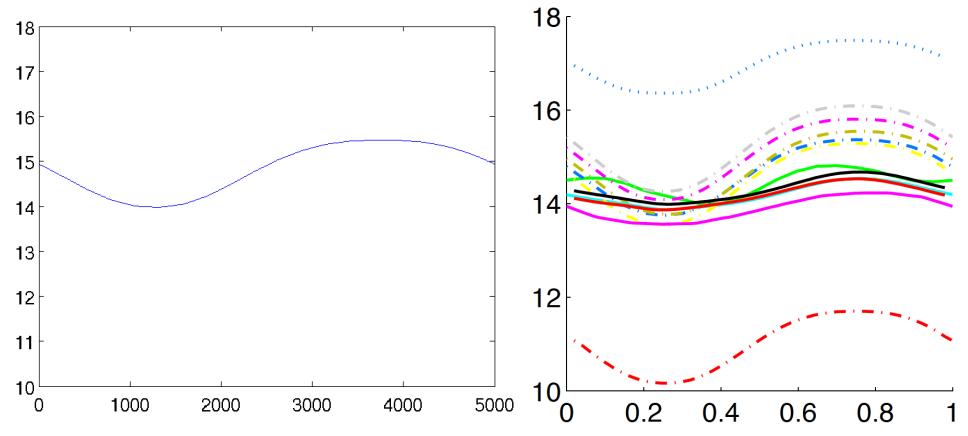
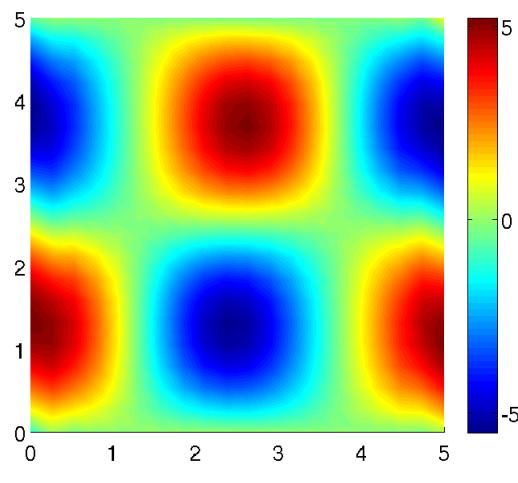


Figure 3.1: Test A geometry

3.1.2 Results

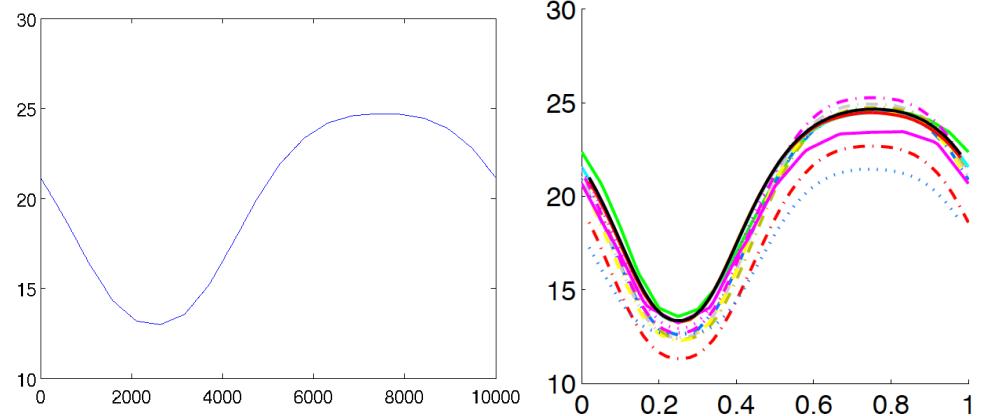
3.1.2.1 5km





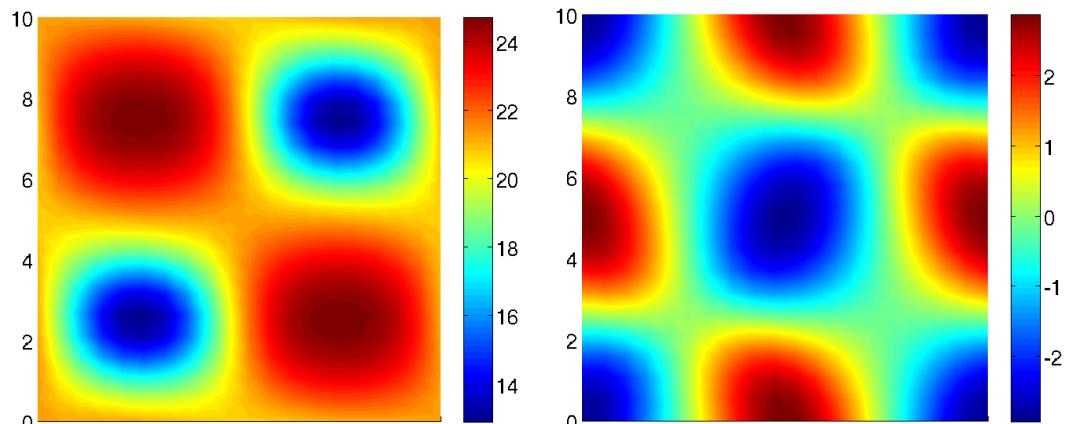
(e) V_z (m/yr) computed by ISSM (Pattyn)

3.1.2.2 10km



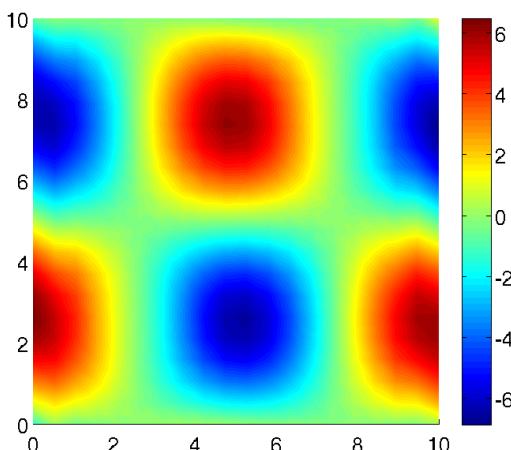
(a) V_x (m/yr) computed by ISSM (Pattyn) on a cross line

(b) V_x (m/yr) comparison



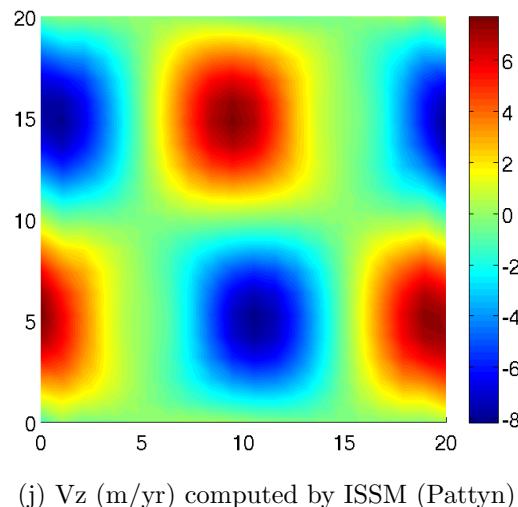
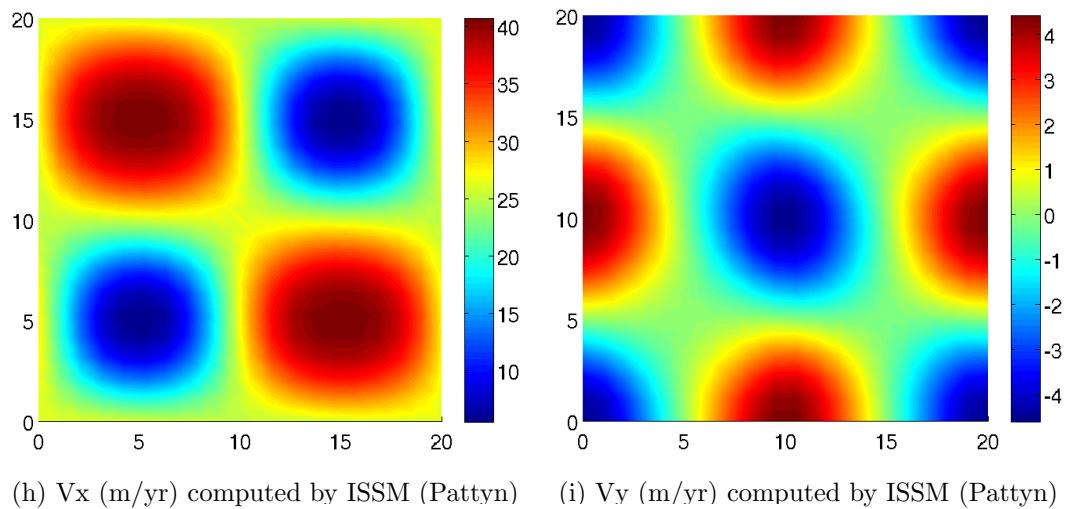
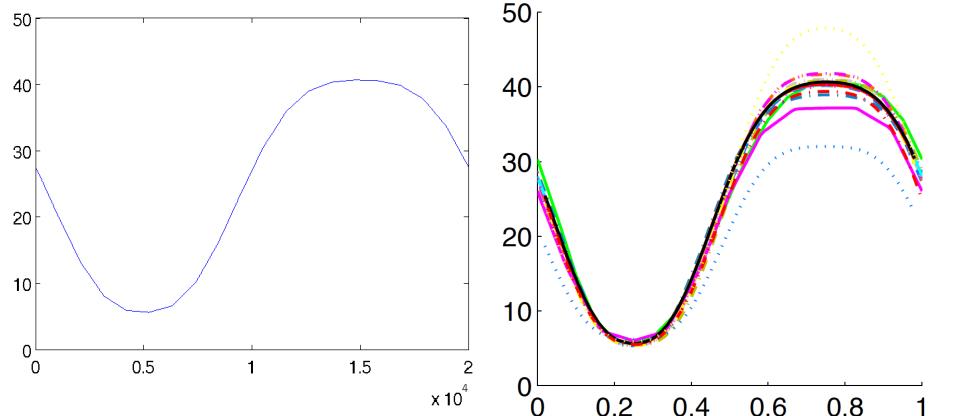
(c) V_x (m/yr) computed by ISSM (Pattyn)

(d) V_y (m/yr) computed by ISSM (Pattyn)

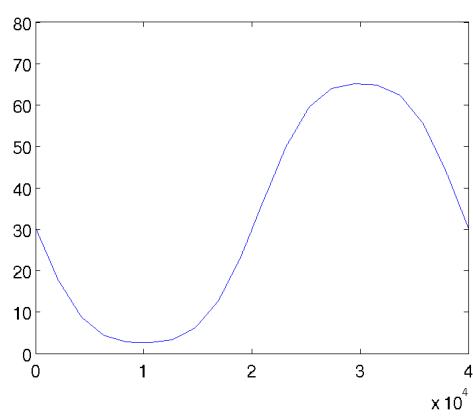


(e) V_z (m/yr) computed by ISSM (Pattyn)

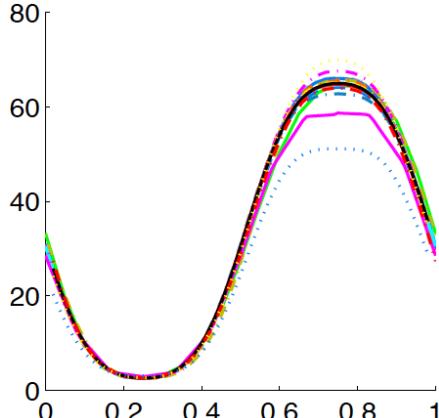
3.1.2.3 20km



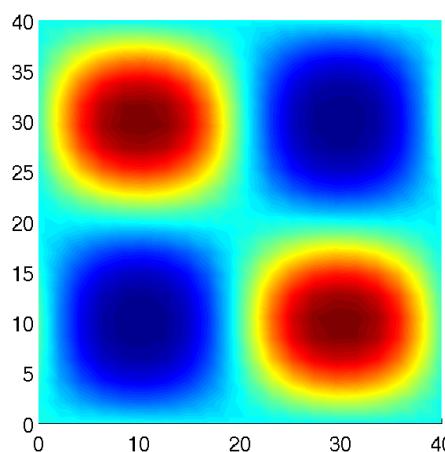
3.1.2.4 40km



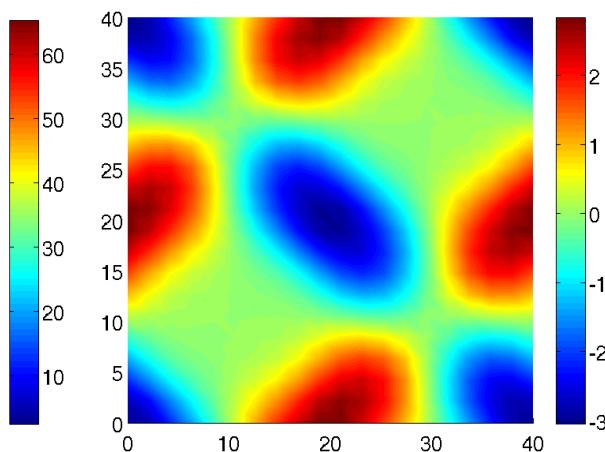
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



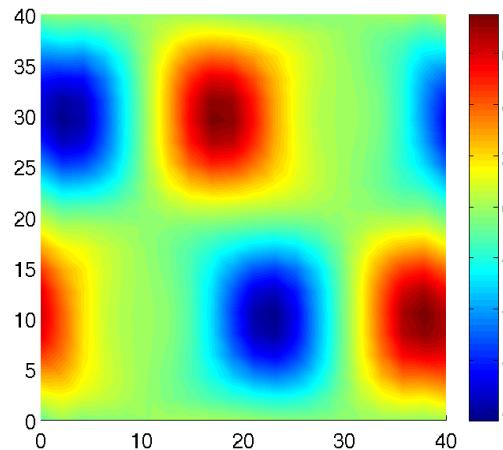
(b) V_x (m/yr) comparison



(c) V_x (m/yr) computed by ISSM (Pattyn)

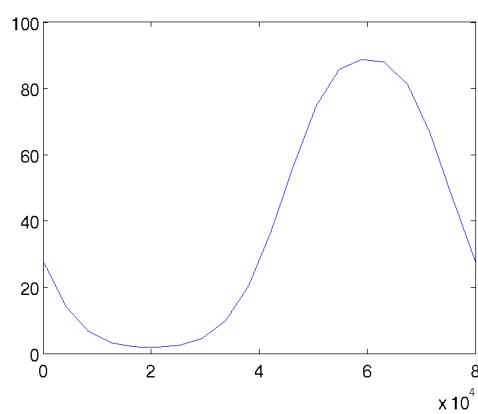


(d) V_y (m/yr) computed by ISSM (Pattyn)

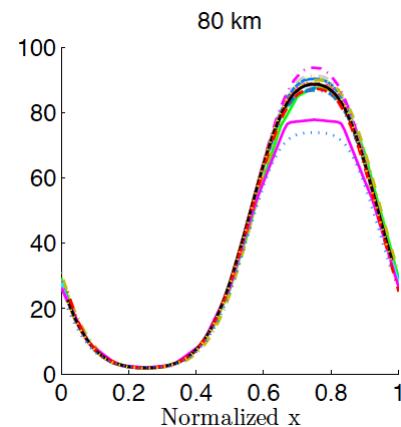


(e) V_z (m/yr) computed by ISSM (Pattyn)

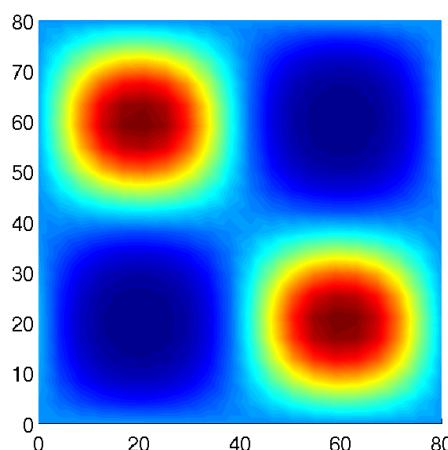
3.1.2.5 80km



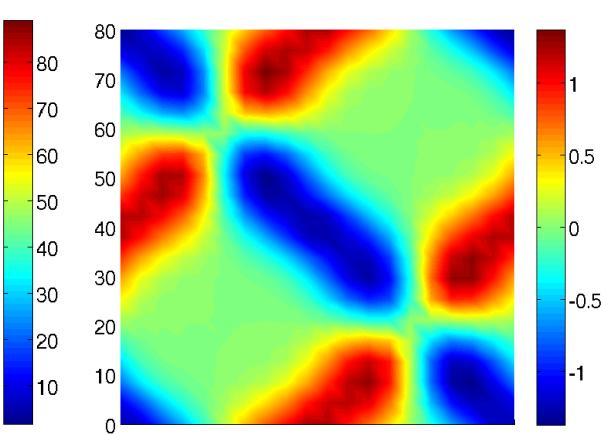
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



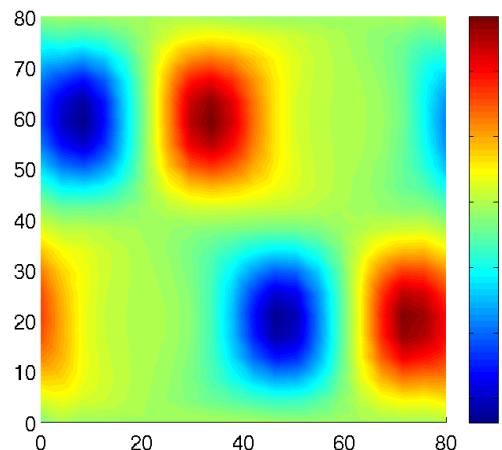
(b) V_x (m/yr) comparison



(c) V_x (m/yr) computed by ISSM (Pattyn)

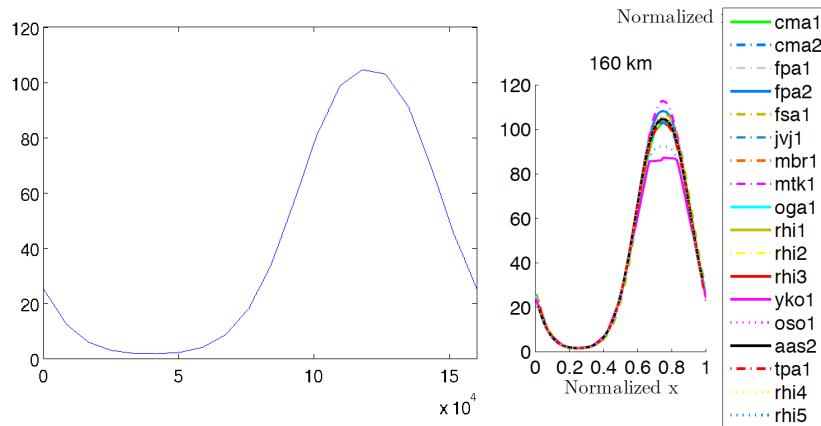


(d) V_y (m/yr) computed by ISSM (Pattyn)

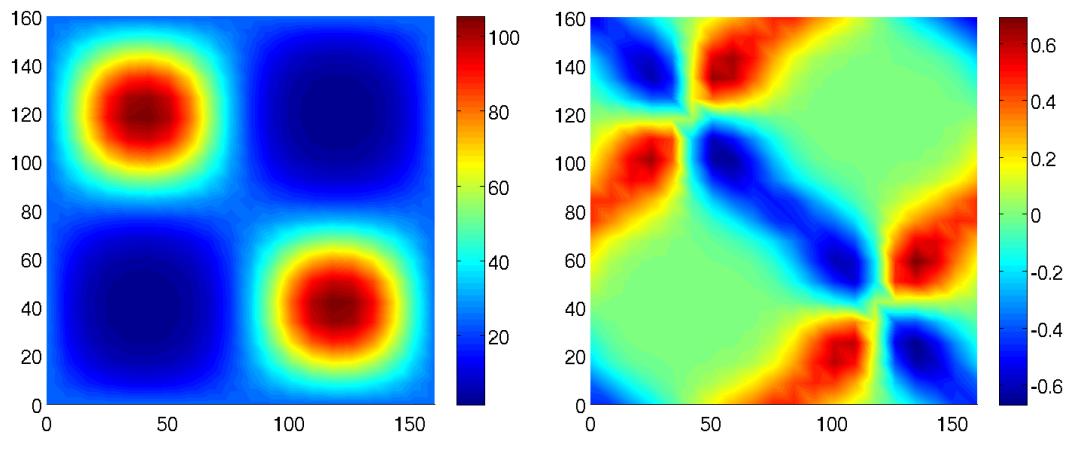


(e) V_z (m/yr) computed by ISSM (Pattyn)

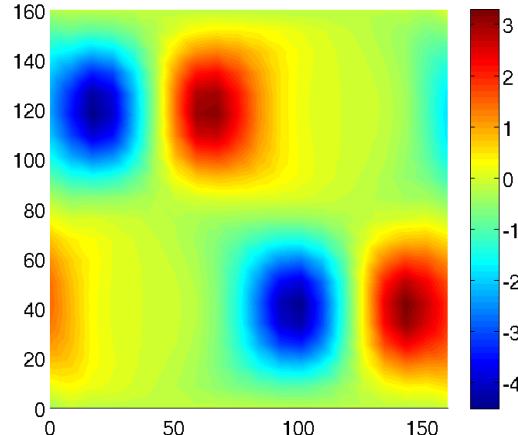
3.1.2.6 160km



(a) V_x (m/yr) computed by ISSM (Pattyn) (b) V_x (m/yr) comparison on a cross line

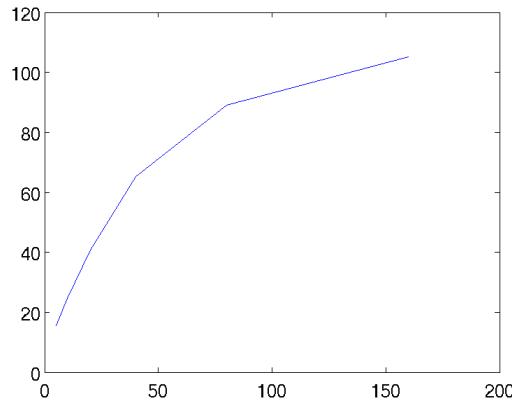


(c) V_x (m/yr) computed by ISSM (Pattyn) (d) V_y (m/yr) computed by ISSM (Pattyn)

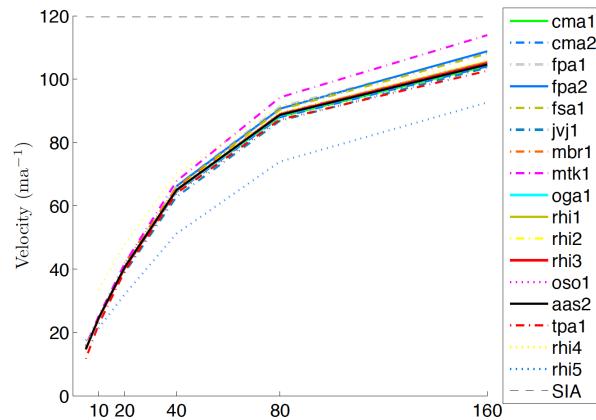


(e) V_z (m/yr) computed by ISSM (Pattyn)

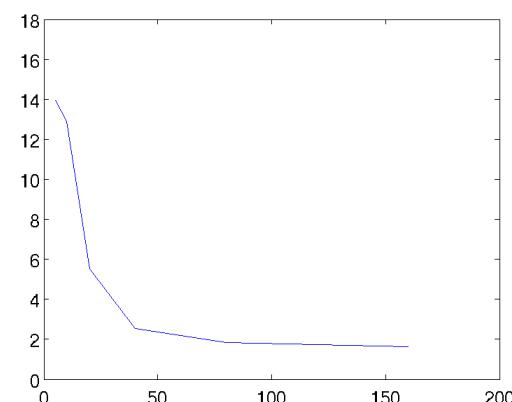
3.1.2.7 global (comparison)



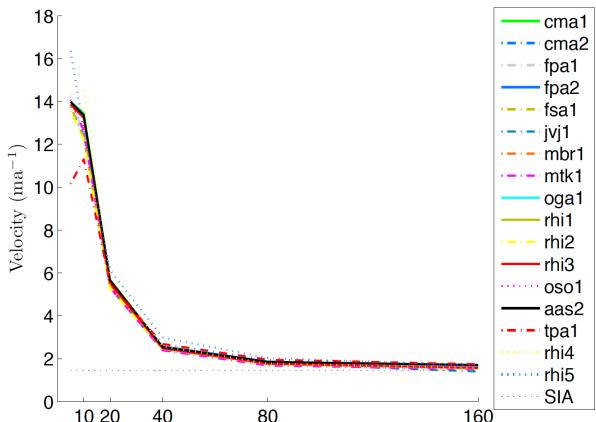
(f) Maximum V_x (m/yr) computed by ISSM (Pattyn)



(g) Maximum V_x (m/yr) comparison



(h) Minimum V_x (m/yr) computed by ISSM (Pattyn)



(i) Minimum V_x (m/yr) comparison

3.2 Test B

3.2.1 Geometry

This is a 2d ice-sheet flow over a rippled bed (flowline). Periodic boundary conditions are applied. The geometry follows:

- surface $s(x, y) = -x \tan(0.5^\circ)$
- bed $b(x, y) = s - 1000 + 500 \sin\left(\frac{2\pi}{L}x\right)$
- $5 \text{ km} \leq L \leq 160 \text{ km}$

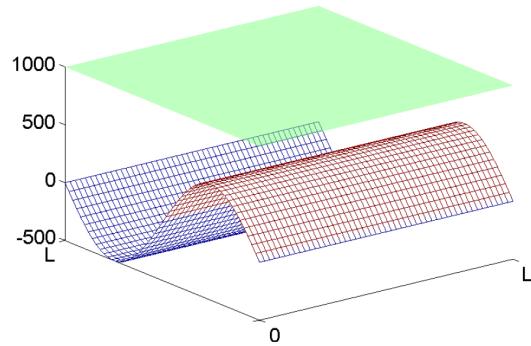
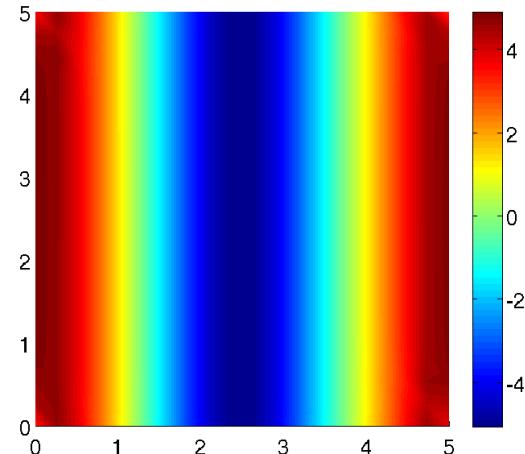
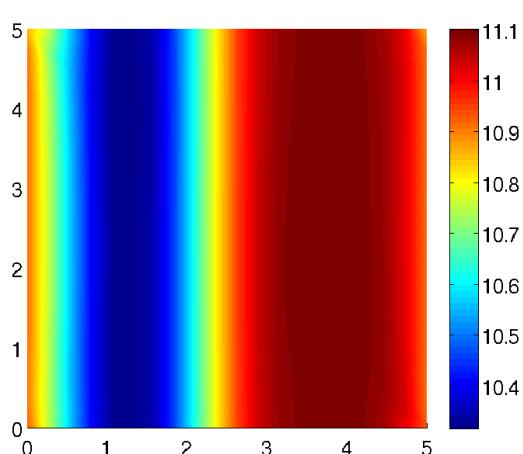
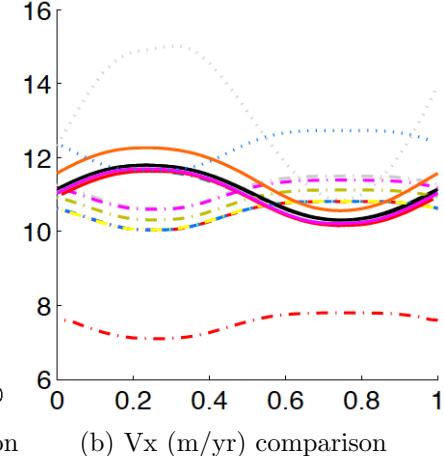
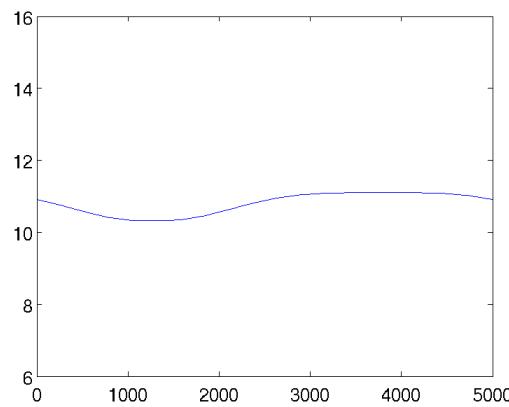


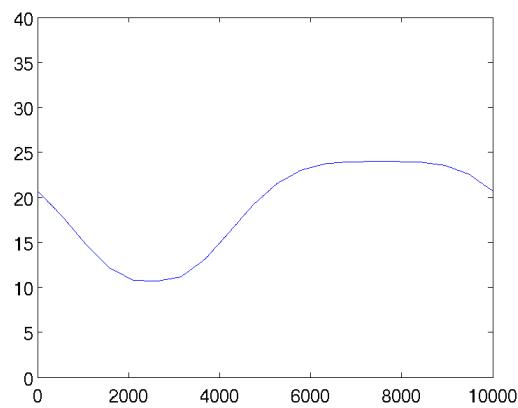
Figure 3.2: Test B geometry

3.2.2 Results

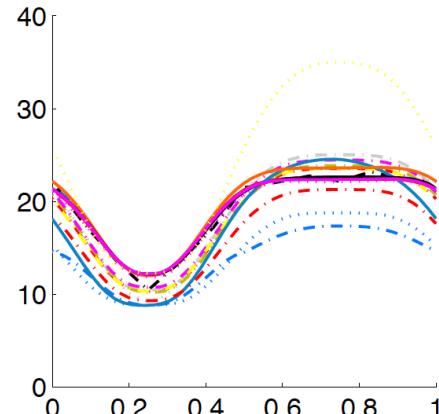
3.2.2.1 5km



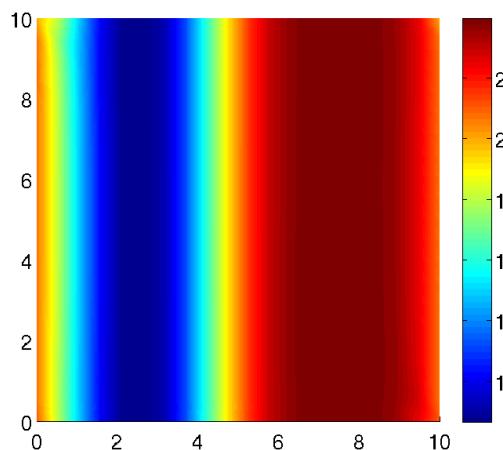
3.2.2.2 10km



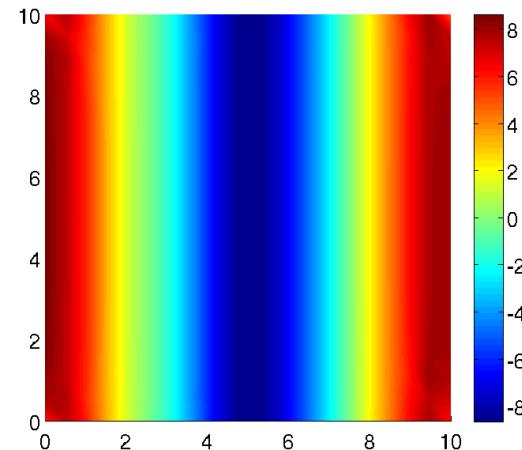
(a) V_x (m/yr) computed by ISSM (Pattyn) on a cross line



(b) V_x (m/yr) comparison

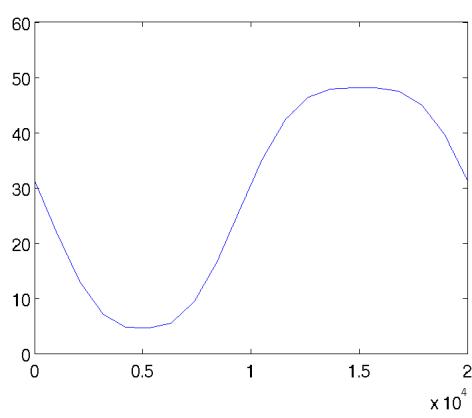


(c) V_x (m/yr) computed by ISSM (Pattyn)

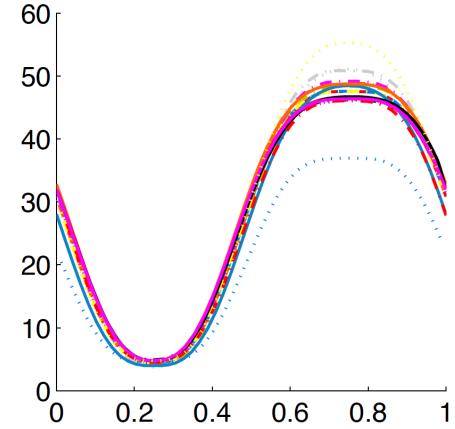


(d) V_z (m/yr) computed by ISSM (Pattyn)

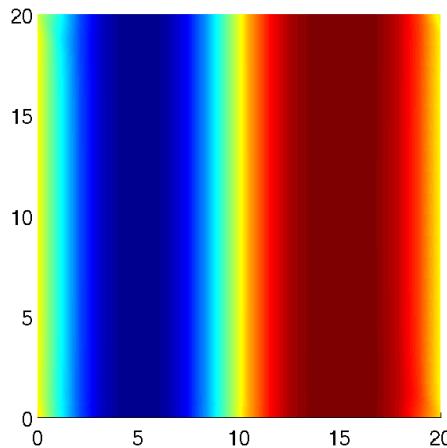
3.2.2.3 20km



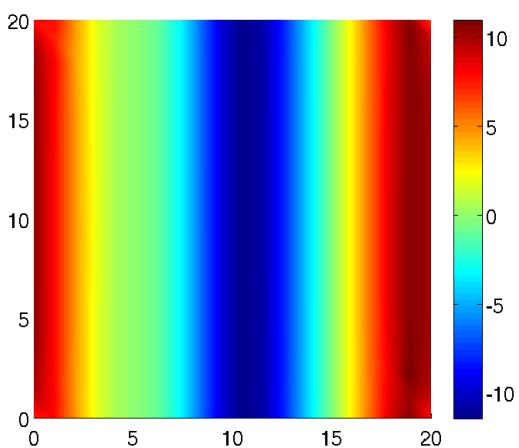
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



(b) V_x (m/yr) comparison

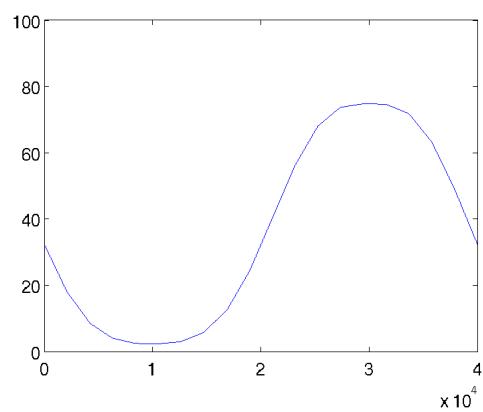


(c) V_x (m/yr) computed by ISSM (Pattyn)

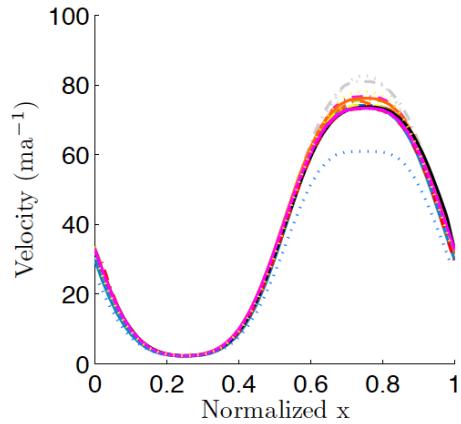


(d) V_z (m/yr) computed by ISSM (Pattyn)

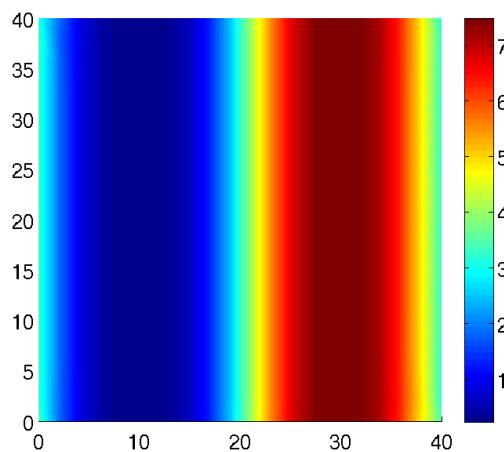
3.2.2.4 40km



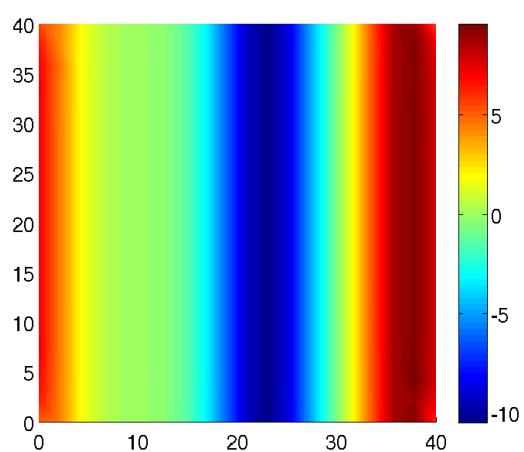
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



(b) V_x (m/yr) comparison

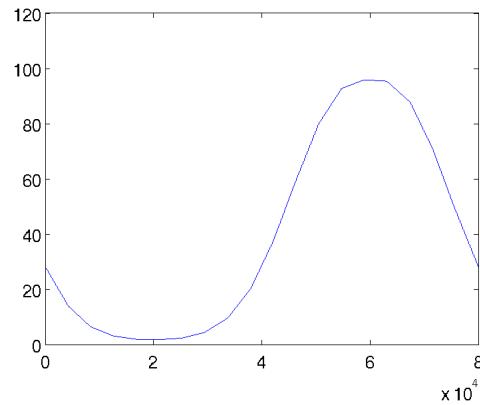


(c) V_x (m/yr) computed by ISSM (Pattyn)

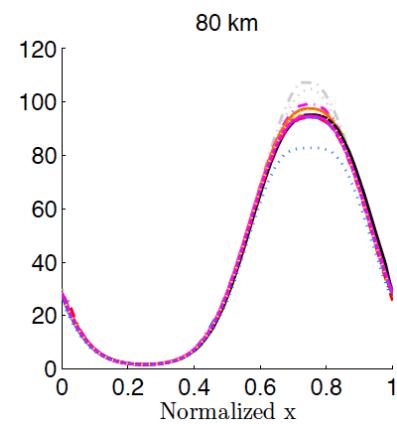


(d) V_z (m/yr) computed by ISSM (Pattyn)

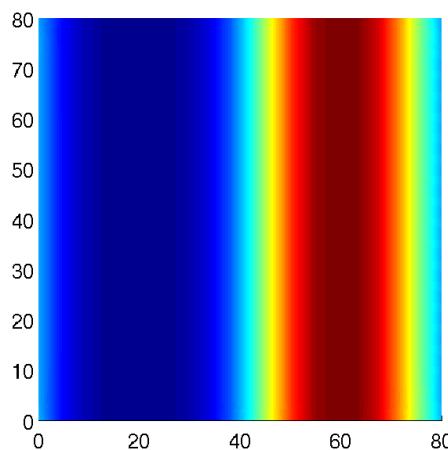
3.2.2.5 80km



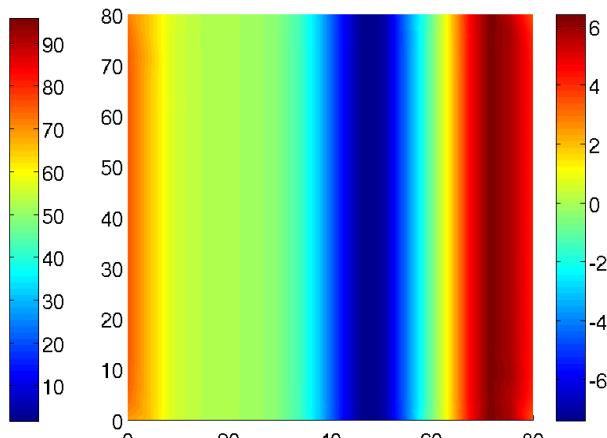
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



(b) V_x (m/yr) comparison

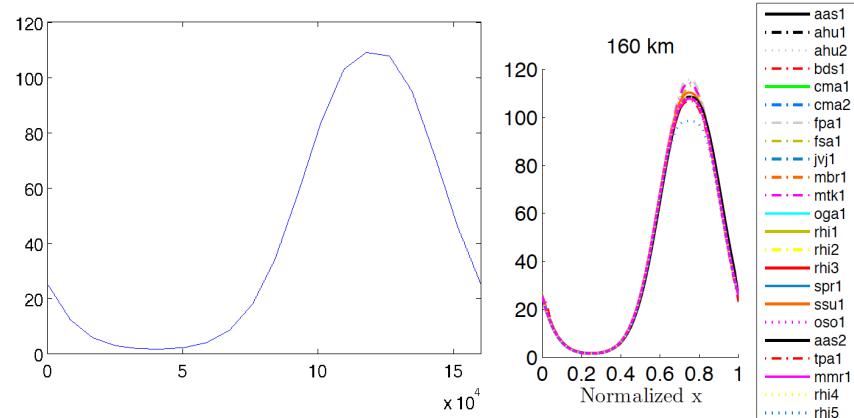


(c) V_x (m/yr) computed by ISSM (Pattyn)

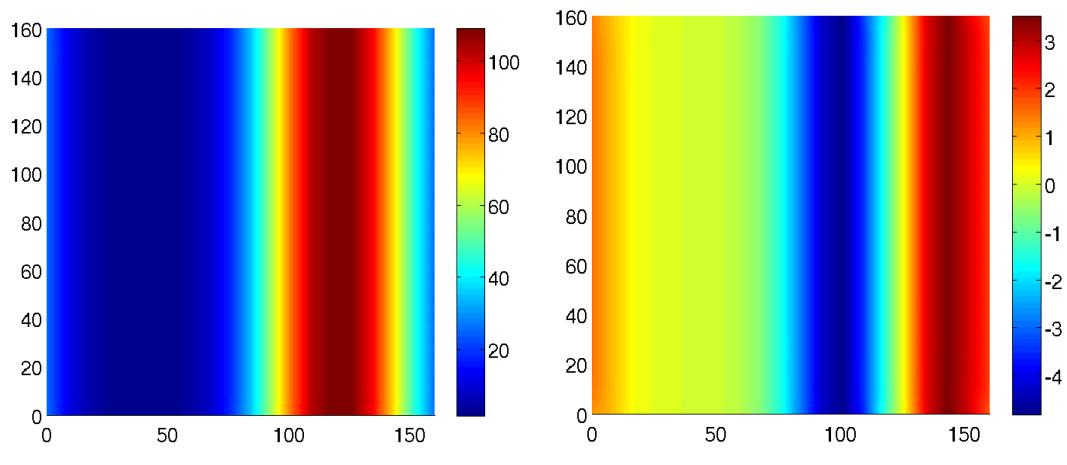


(d) V_z (m/yr) computed by ISSM (Pattyn)

3.2.2.6 160km

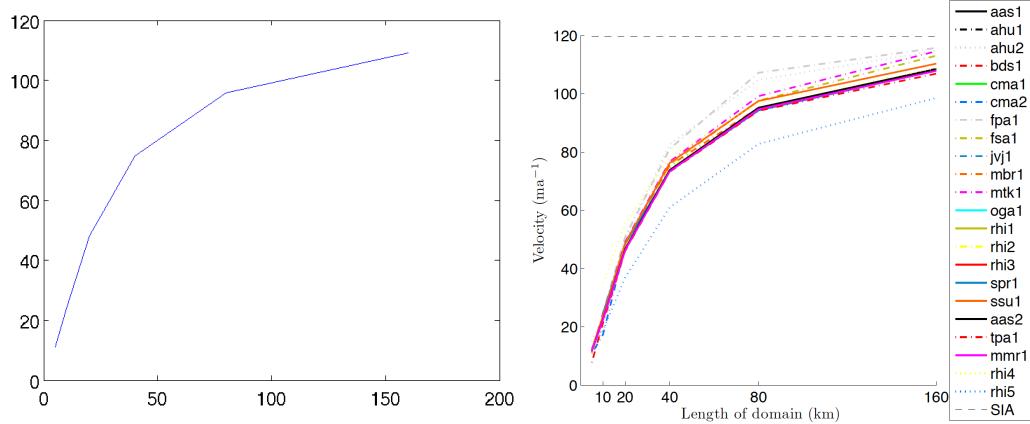


(a) V_x (m/yr) computed by ISSM (Pattyn) (b) V_x (m/yr) comparison
on a cross line



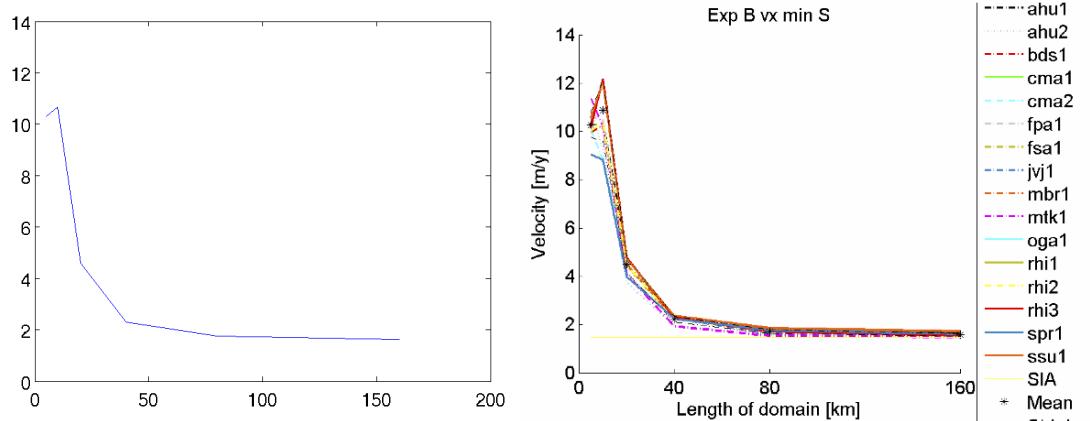
(c) V_x (m/yr) computed by ISSM (Pattyn) (d) V_z (m/yr) computed by ISSM (Pattyn)

3.2.2.7 global (comparison)



(e) Maximum Vx (m/yr) computed by ISSM (Pattyn)

(f) Maximum Vx (m/yr) comparison



(g) Minimum Vx (m/yr) computed by ISSM (Pattyn)

(h) Minimum Vx (m/yr) comparison

3.3 Test C (detailed)

3.3.1 Geometry

This is a 3d ice-stream flow over a slippery bed. Periodic boundary conditions are applied. The geometry follows:

- surface $s(x, y) = -x \tan(0.1^\circ)$
- bed $b(x, y) = s - 1000$
- sliding $\alpha^2(x, y) = 1000 + 1000 \sin\left(\frac{2\pi}{L}x\right) \sin\left(\frac{2\pi}{L}y\right)$
- $5 \text{ km} \leq L \leq 160 \text{ km}$

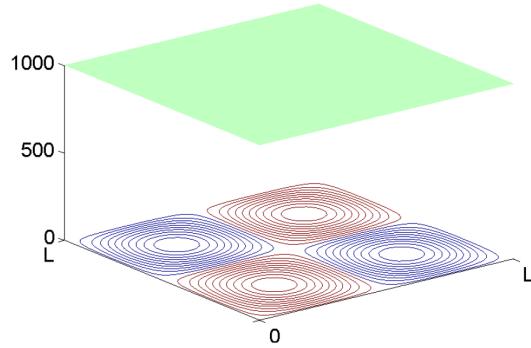
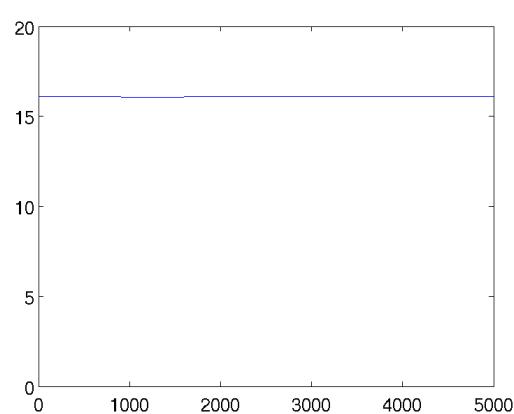


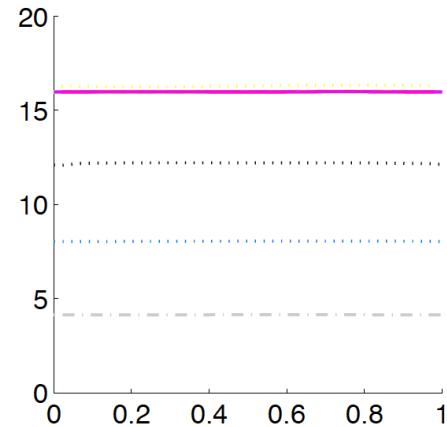
Figure 3.3: Test C geometry

3.3.2 Results

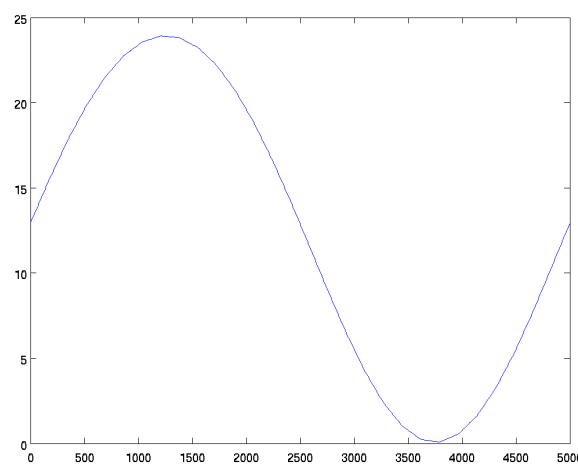
3.3.2.1 5km



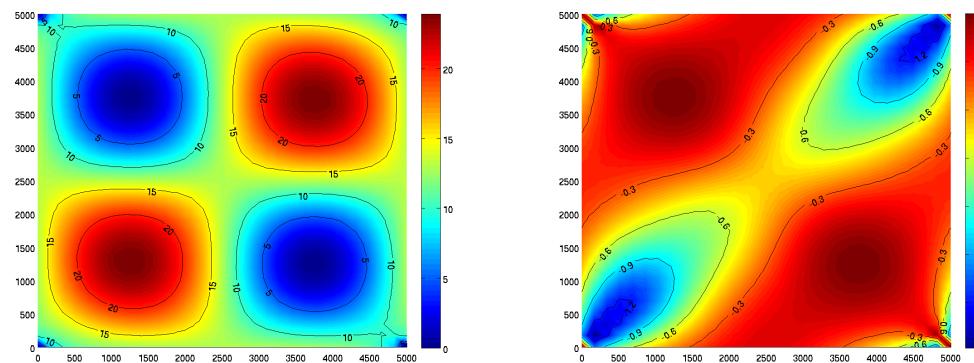
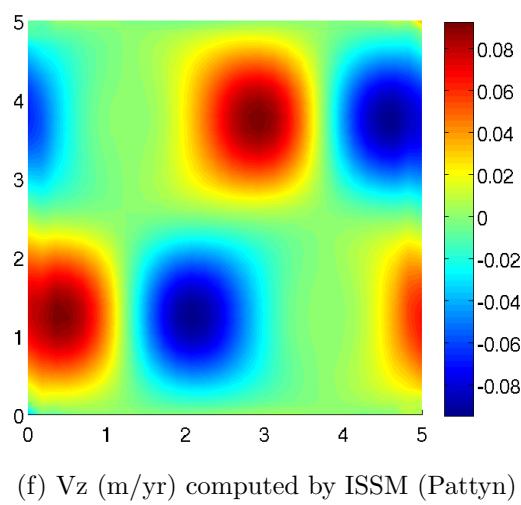
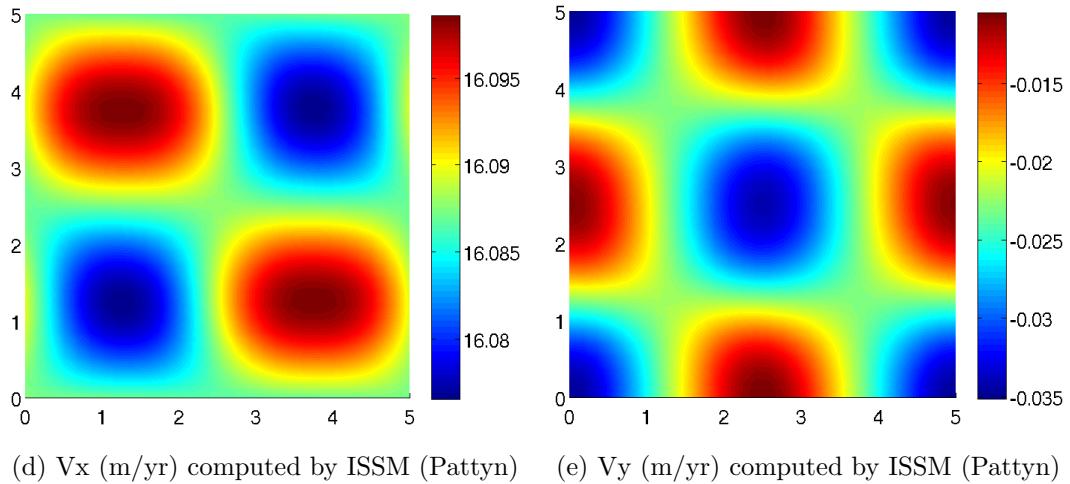
(a) V_x (m/yr) computed by ISSM (Pattyn) on a cross line



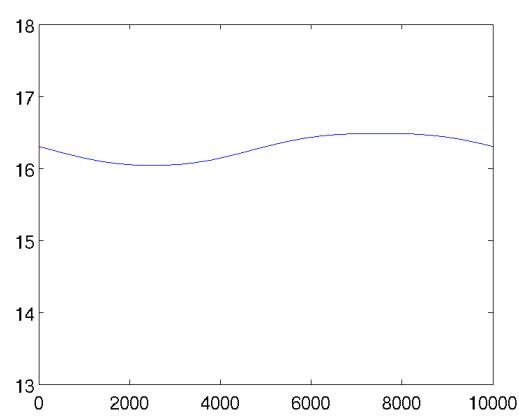
(b) V_x (m/yr) comparison



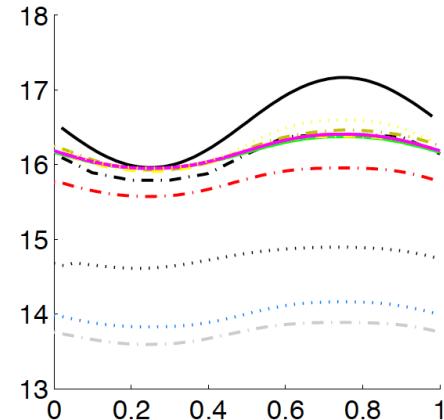
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on a cross line



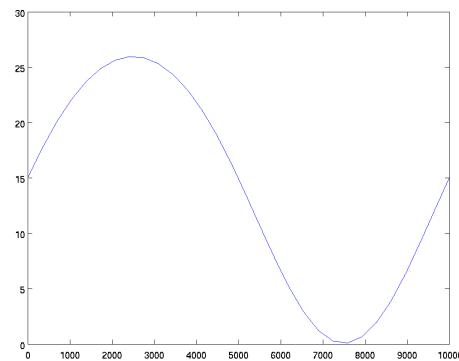
3.3.2.2 10km



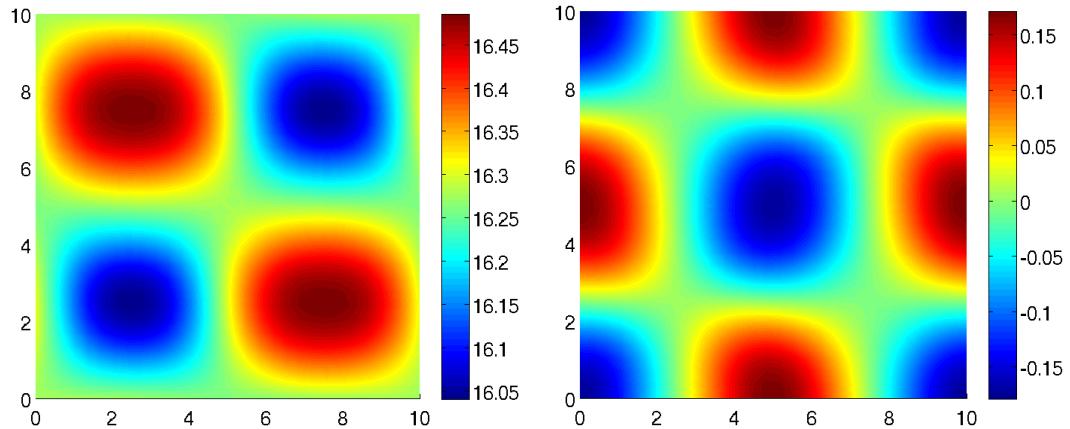
(a) V_x (m/yr) computed by ISSM (Pattyn) on
a cross line



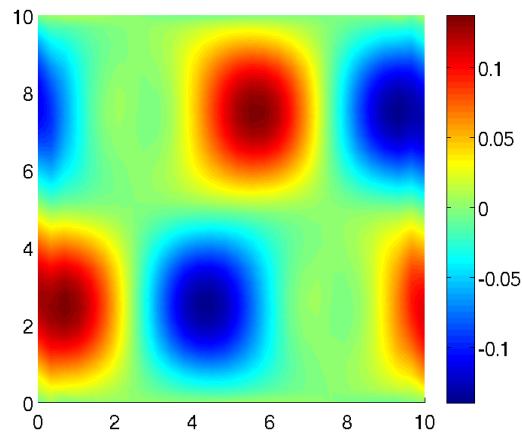
(b) V_x (m/yr) comparison



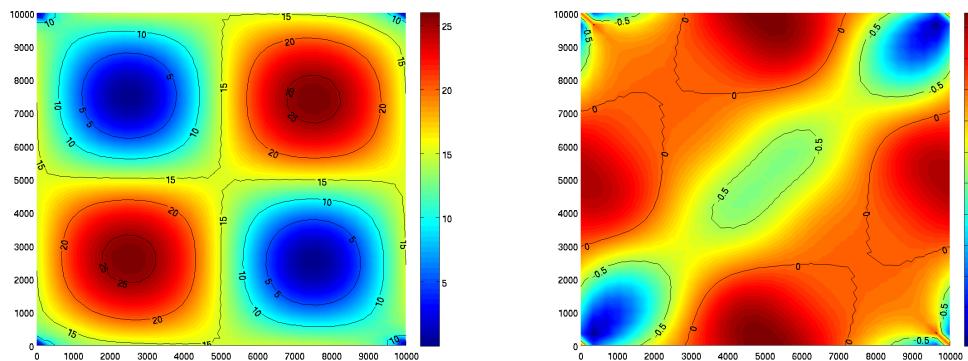
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on
a cross line



(d) V_x (m/yr) computed by ISSM (Pattyn) (e) V_y (m/yr) computed by ISSM (Pattyn)

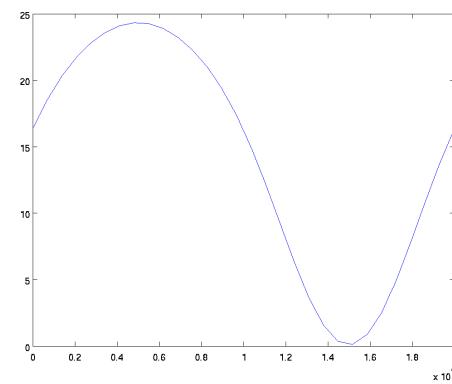
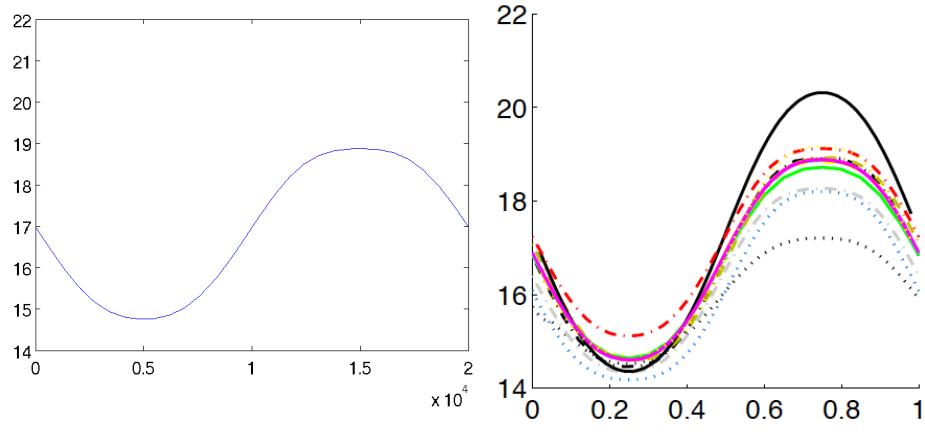


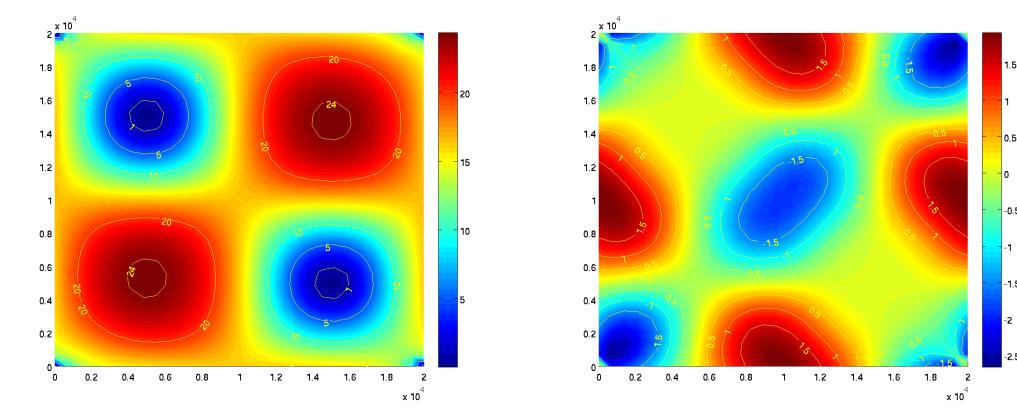
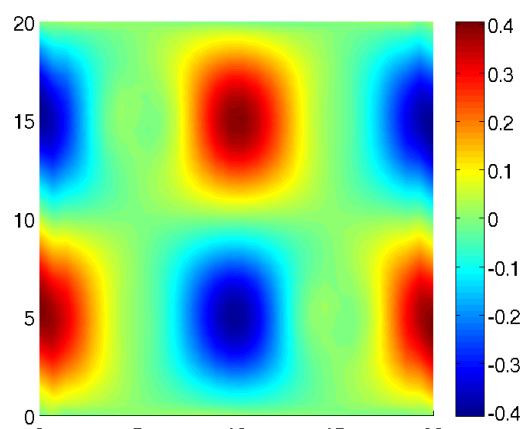
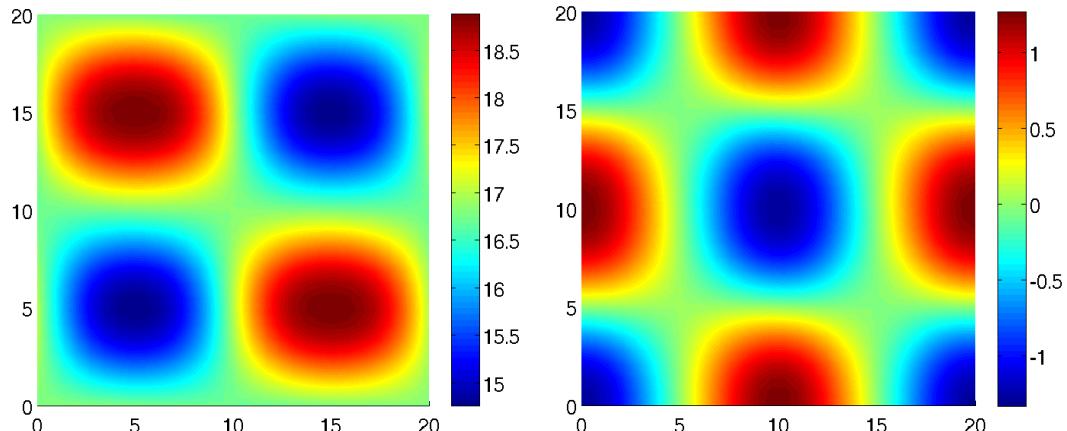
(f) V_z (m/yr) computed by ISSM (Pattyn)



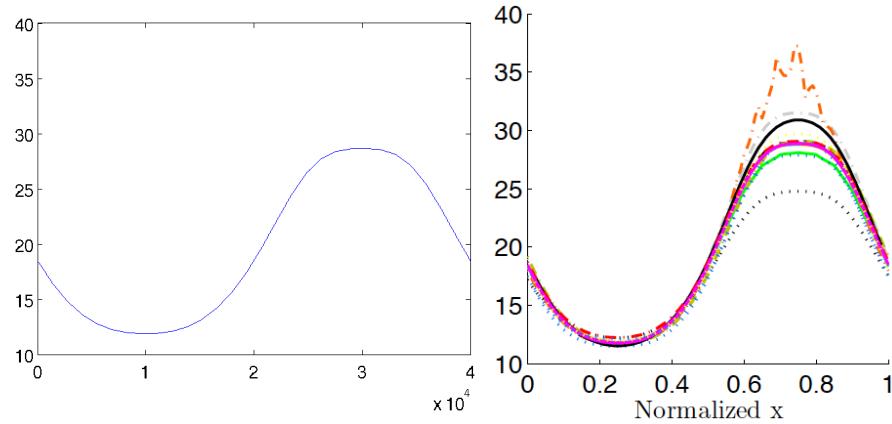
(g) T_{xz} (kPa) computed by ISSM (Pattyn) (h) T_{yz} (kPa) computed by ISSM (Pattyn)

3.3.2.3 20km

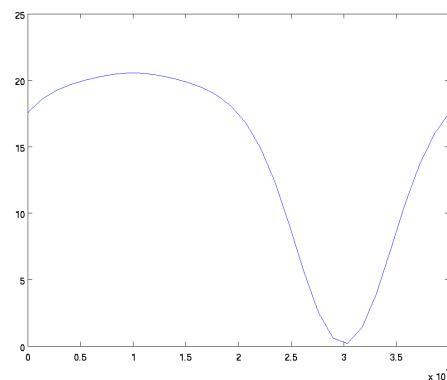




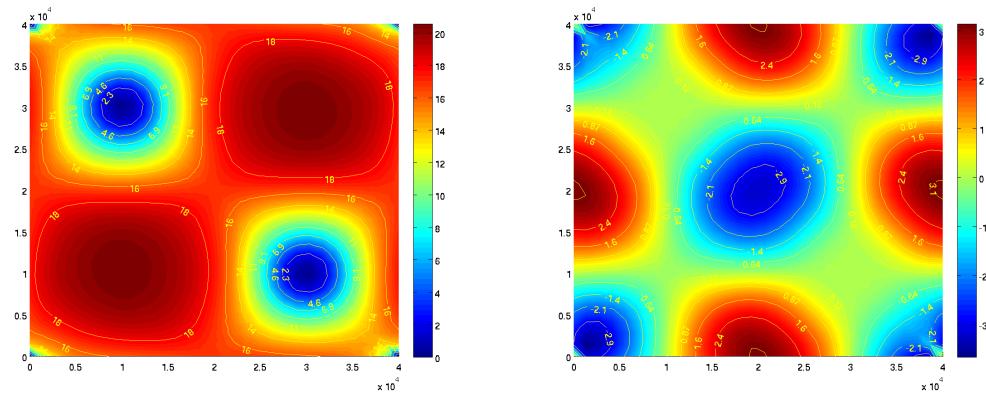
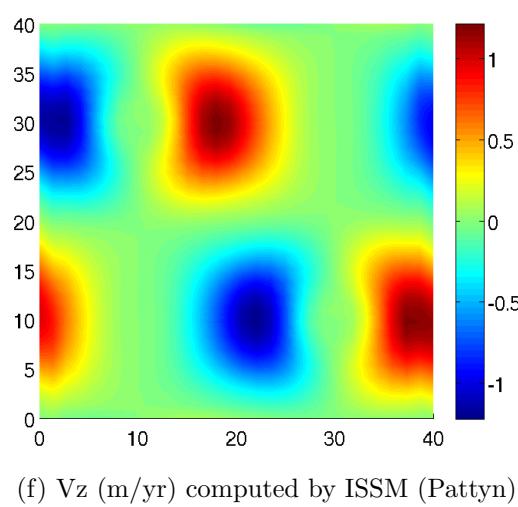
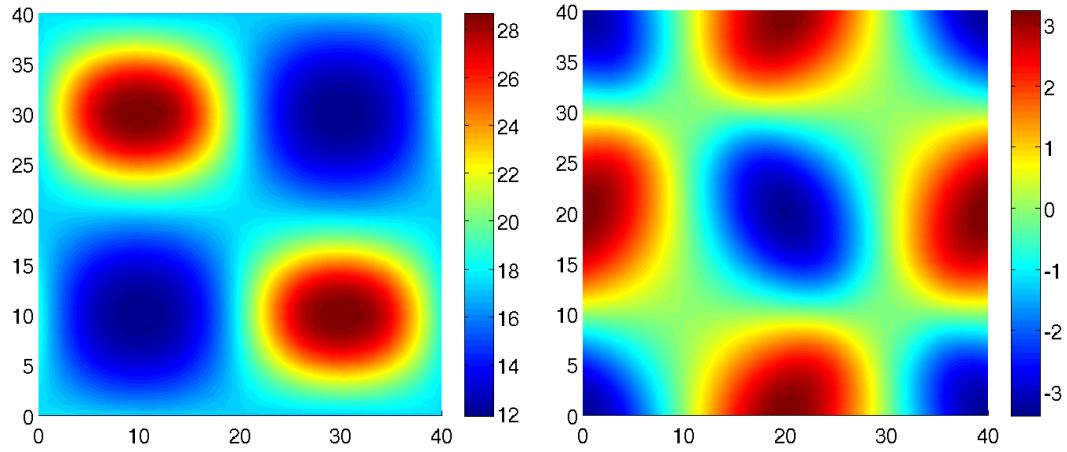
3.3.2.4 40km



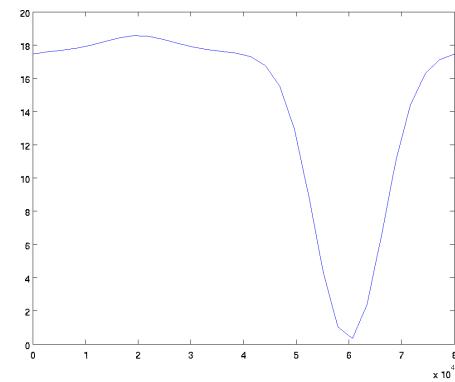
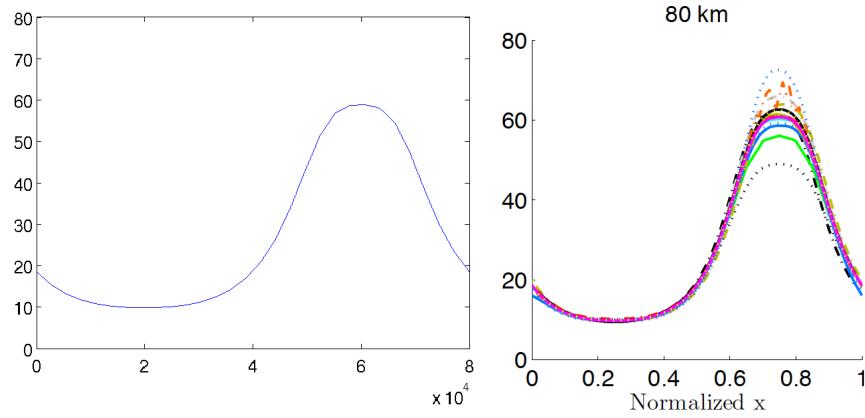
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line (b) V_x (m/yr) comparison



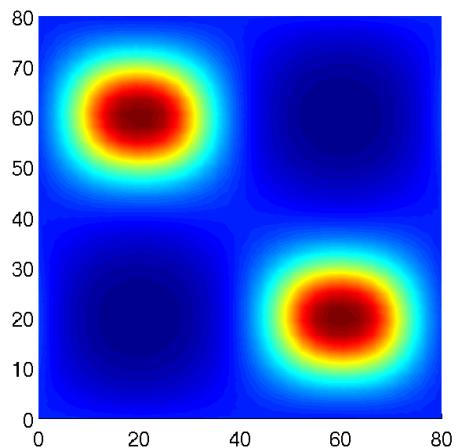
(c) T_{xz} (kPa) computed by ISSM (Pattyn)
on a cross line



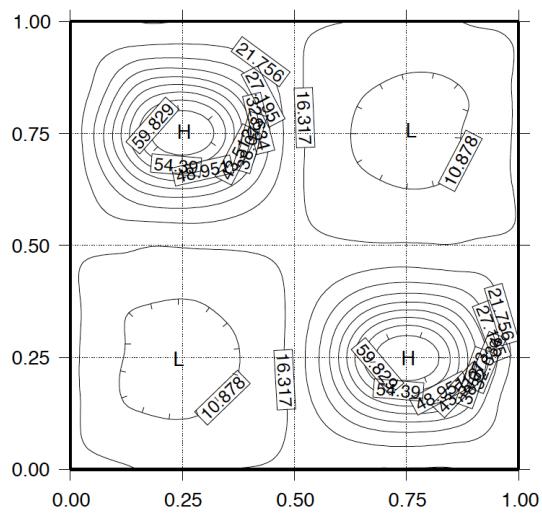
3.3.2.5 80km



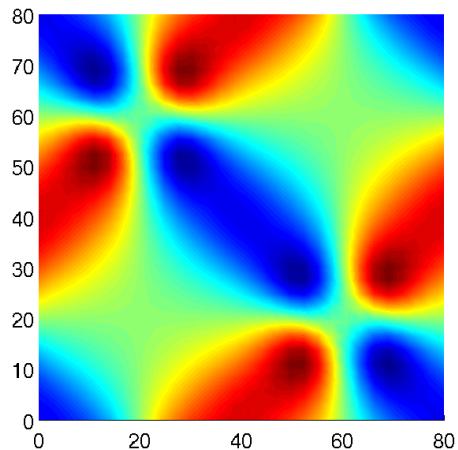
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on a cross line



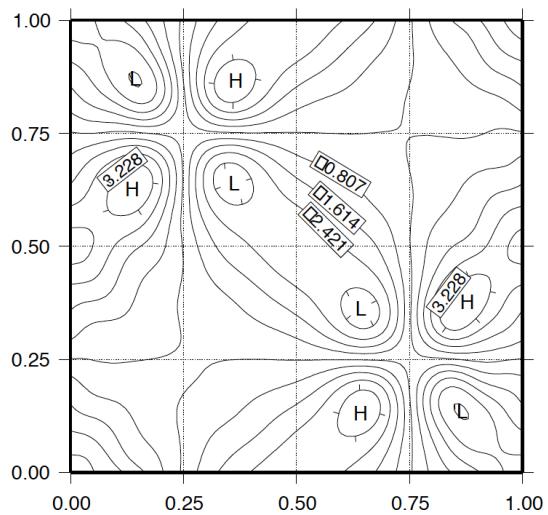
(d) V_x (m/yr) computed by ISSM (Pattyn)



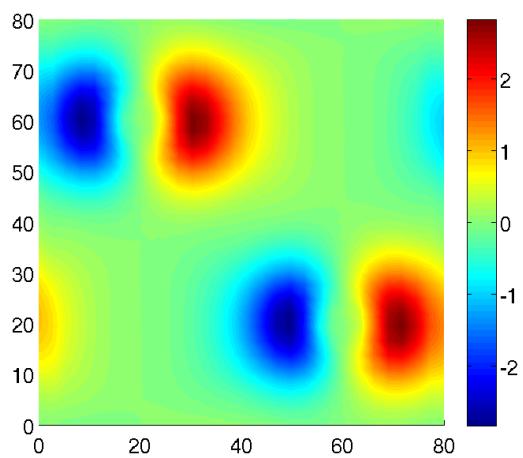
(e) V_x (m/yr) comparison



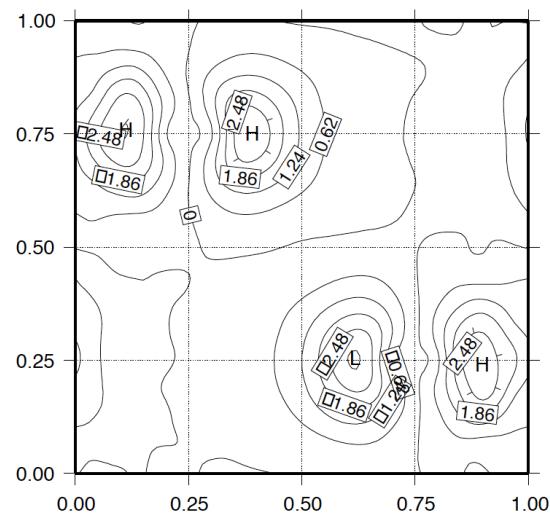
(f) V_y (m/yr) computed by ISSM (Pattyn)



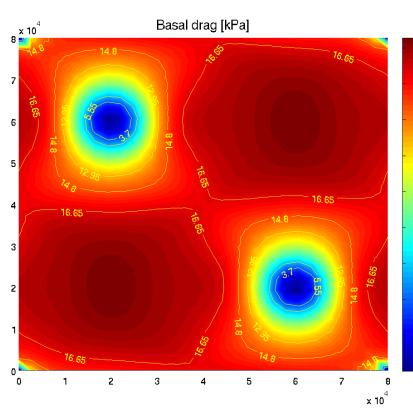
(g) V_y (m/yr) comparison



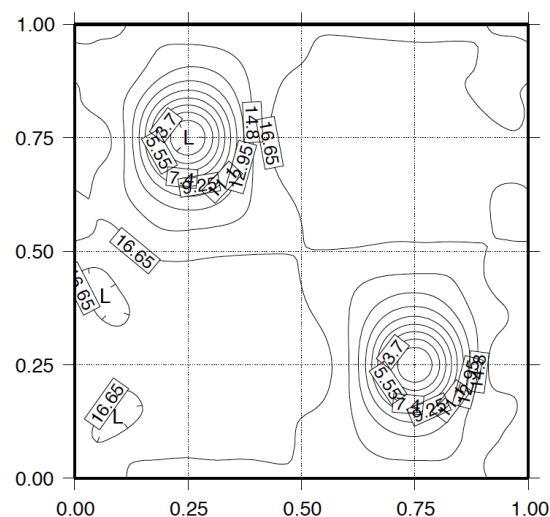
(h) V_z (m/yr) computed by ISSM (Pattyn)



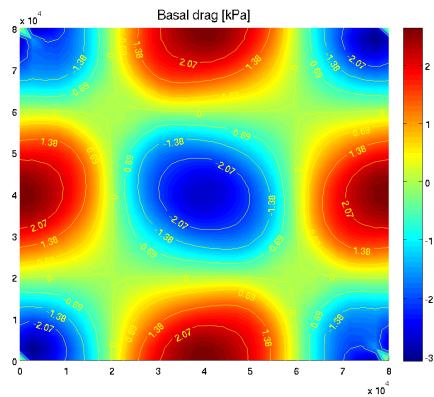
(i) V_z (m/yr) compatison



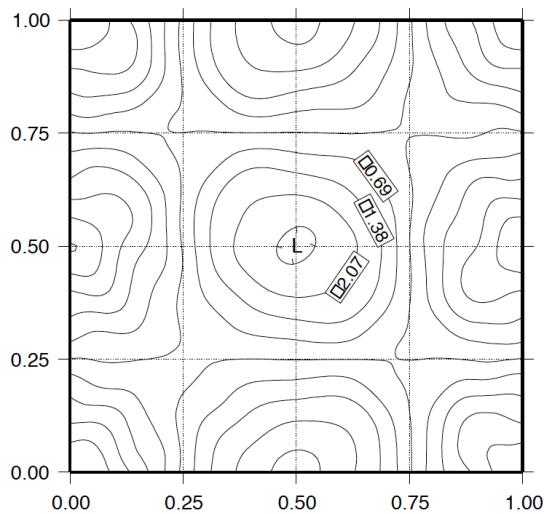
(j) T_{xz} (kPa) computed by ISSM (Pattyn)



(k) T_{xz} (kPa) compatison

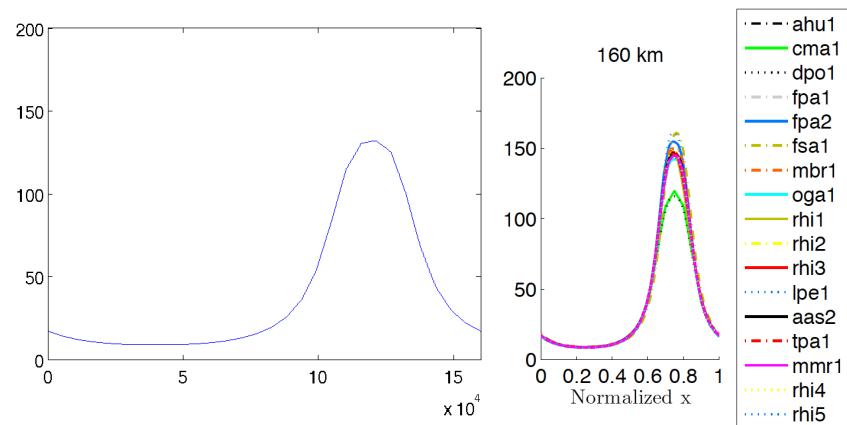


(l) Tyz (kPa) computed by ISSM (Pattyn)

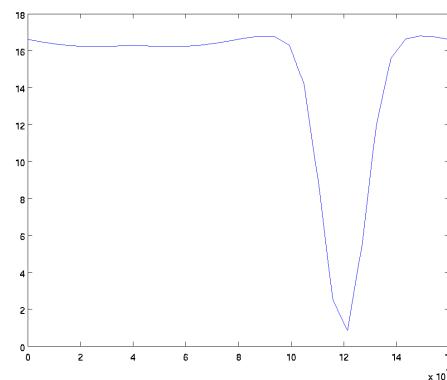


(m) Tyz (kPa) compatision

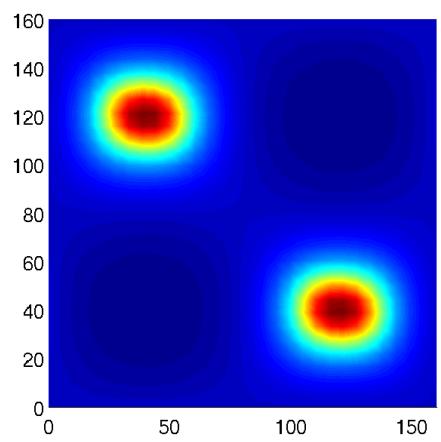
3.3.2.6 160km



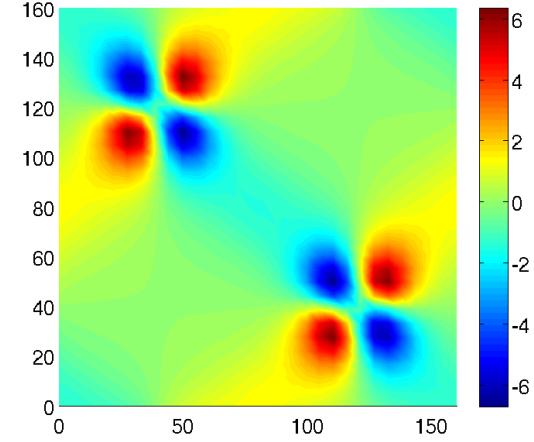
(a) V_x (m/yr) computed by ISSM (Pattyn) (b) V_x (m/yr) comparison
on a cross line



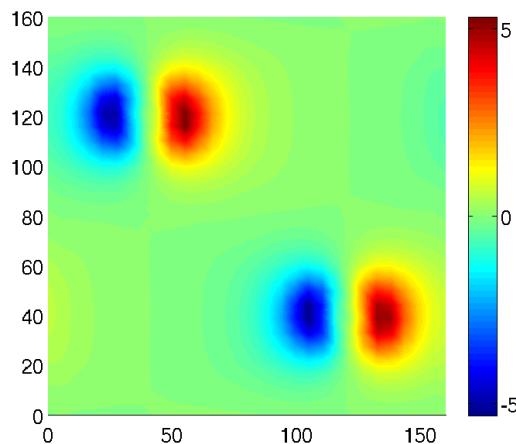
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on
a cross line



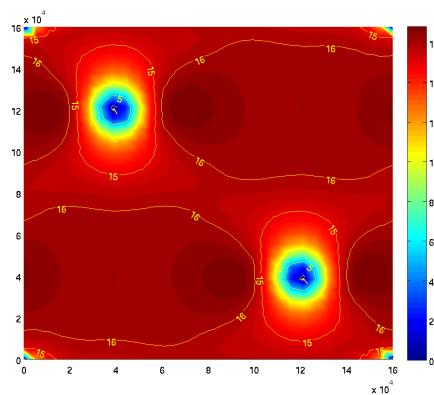
(d) V_x (m/yr) computed by ISSM (Pattyn)



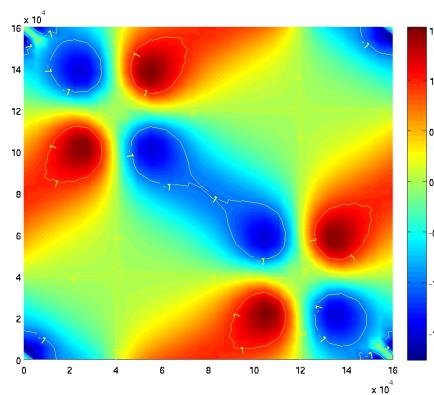
(e) V_y (m/yr) computed by ISSM (Pattyn)



(f) V_z (m/yr) computed by ISSM (Pattyn)

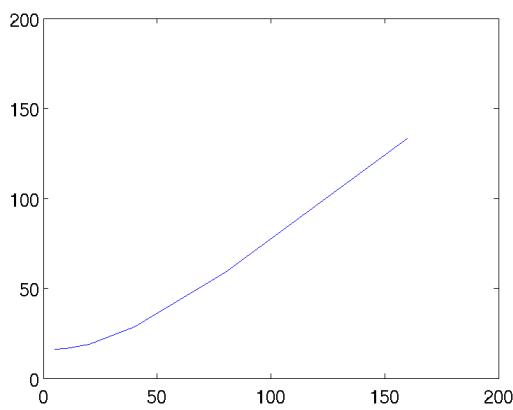


(g) T_{xz} (kPa) computed by ISSM (Pattyn)

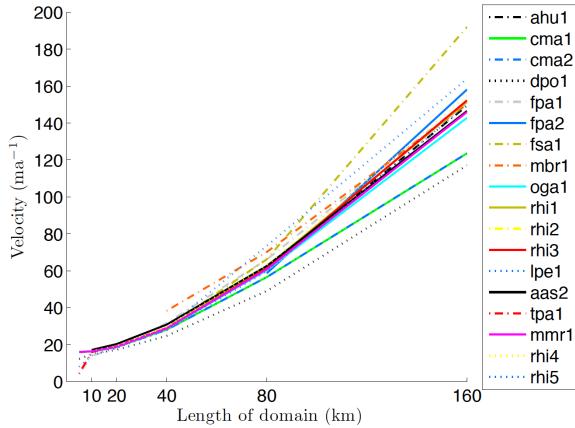


(h) T_{yz} (kPa) computed by ISSM (Pattyn)

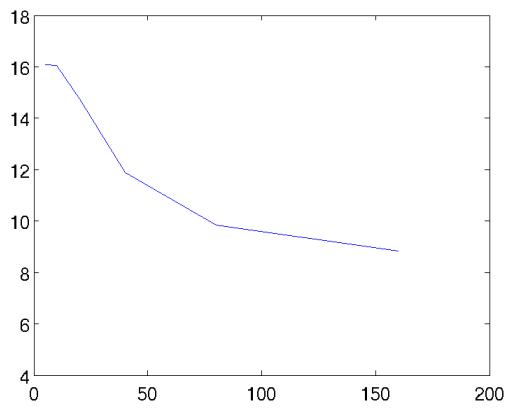
3.3.2.7 global (comparison)



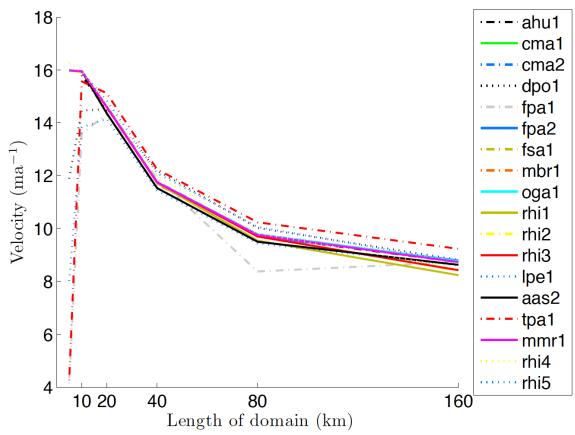
(i) Maximum V_x (m/yr) computed by ISSM (Pattyn)



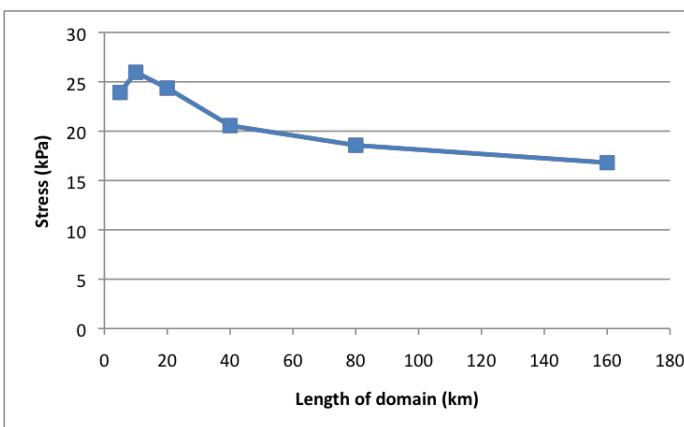
(j) Maximum V_x (m/yr) comparison



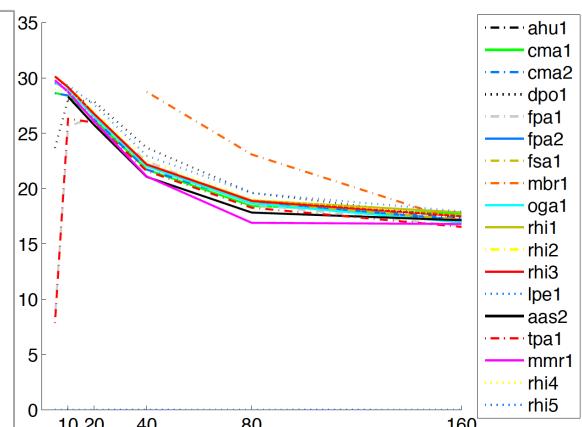
(k) Minimum V_x (m/yr) computed by ISSM (Pattyn)



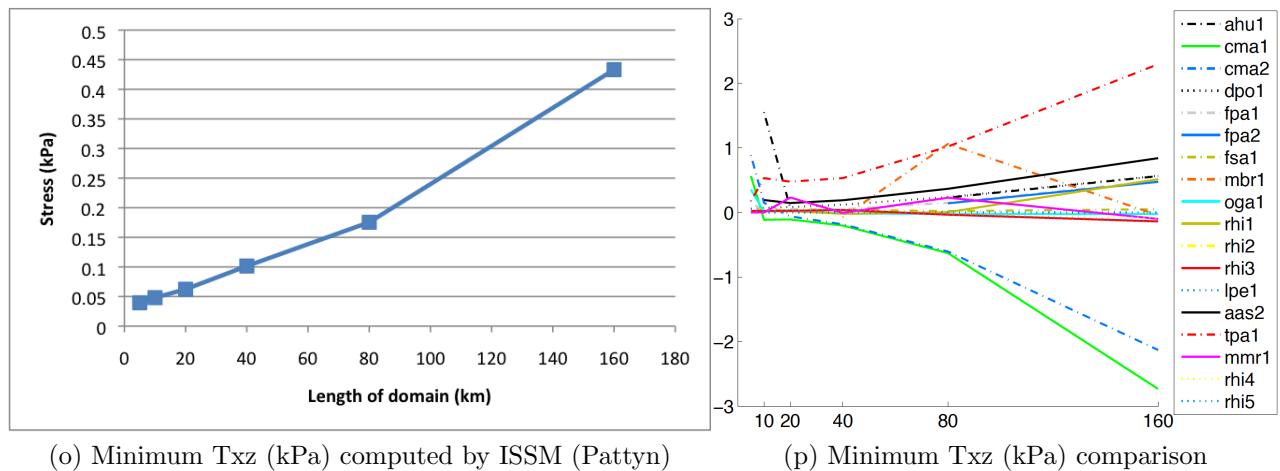
(l) Minimum V_x (m/yr) comparison



(m) Maximum T_{xz} (kPa) computed by ISSM (Pattyn)



(n) Maximum T_{xz} (kPa) comparison



3.4 Test D

3.4.1 Geometry

This is a 2d ice-stream flow over a slippery bed (flowline). Periodic boundary conditions are applied. The geometry follows:

- surface $s(x, y) = -x \tan(0.1^\circ)$
- bed $b(x, y) = s - 1000$
- sliding $\alpha^2(x, y) = 1000 + 1000 \sin\left(\frac{2\pi}{L}x\right)$
- $5 \text{ km} \leq L \leq 160 \text{ km}$

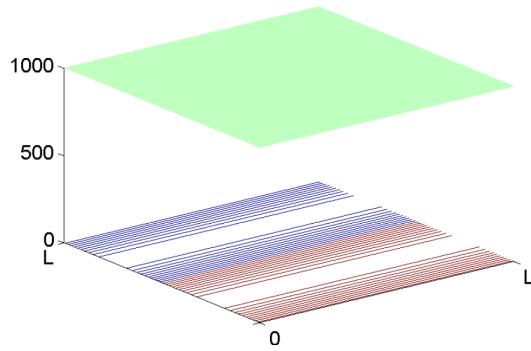
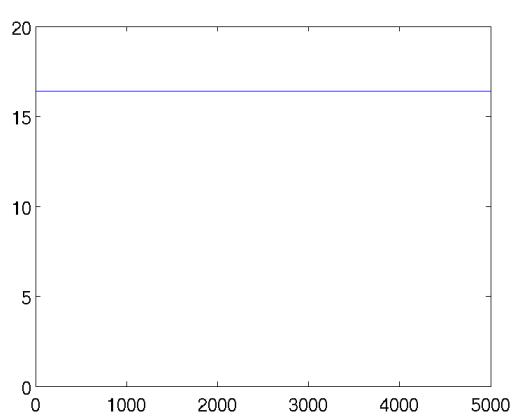


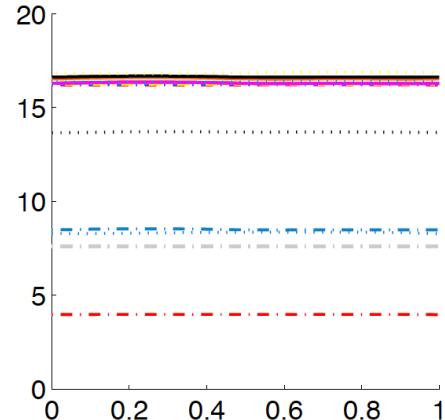
Figure 3.4: Test D geometry

3.4.2 Results

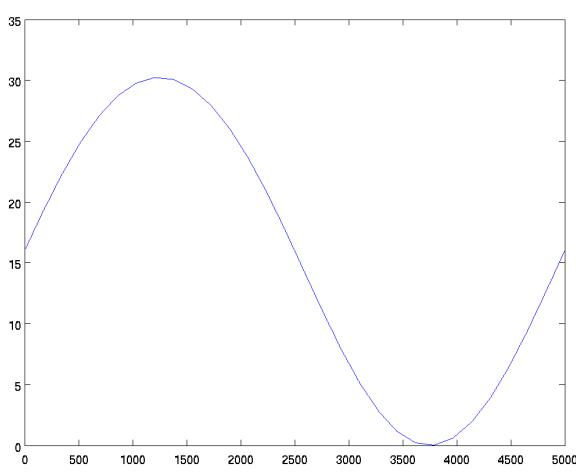
3.4.2.1 5km



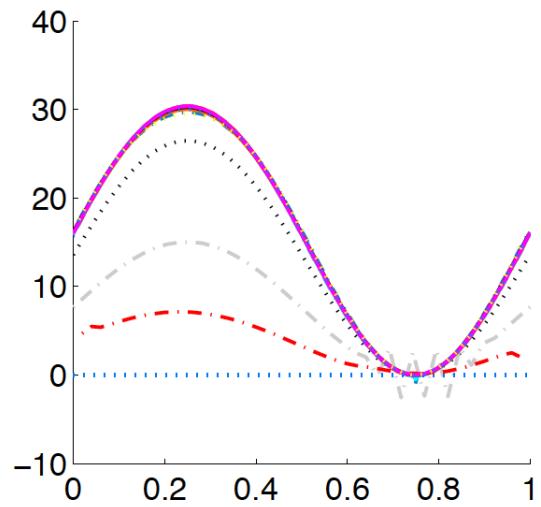
(a) V_x (m/yr) computed by ISSM (Pattyn) on a cross line



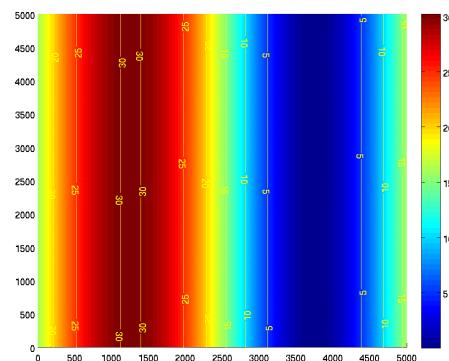
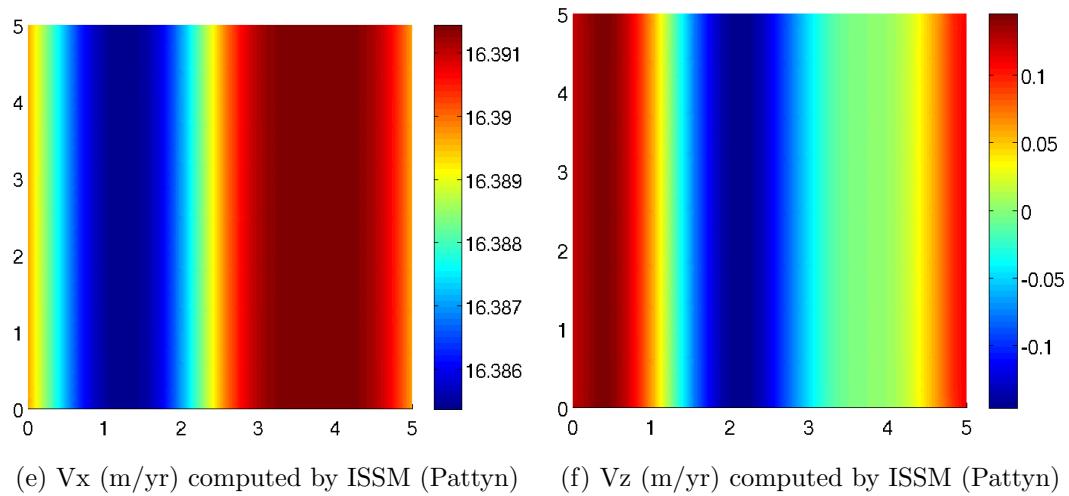
(b) V_x (m/yr) comparison



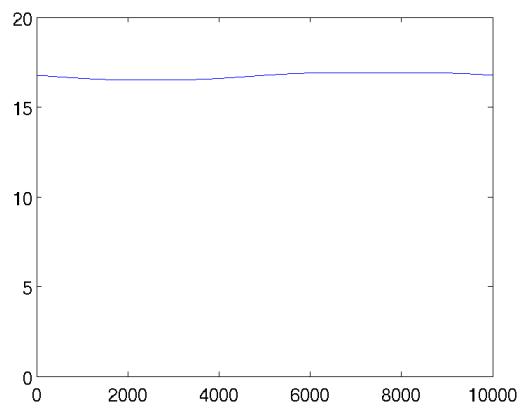
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on a cross line



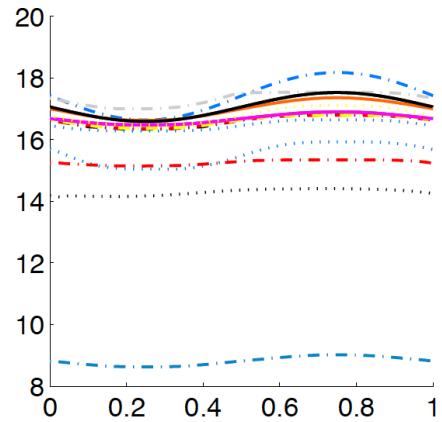
(d) T_{xz} (kPa) comparison



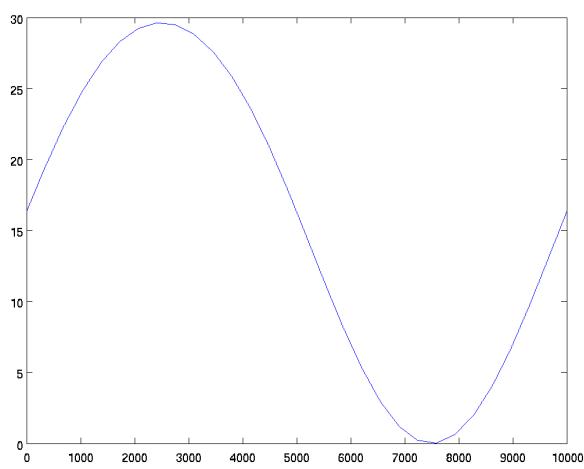
3.4.2.2 10km



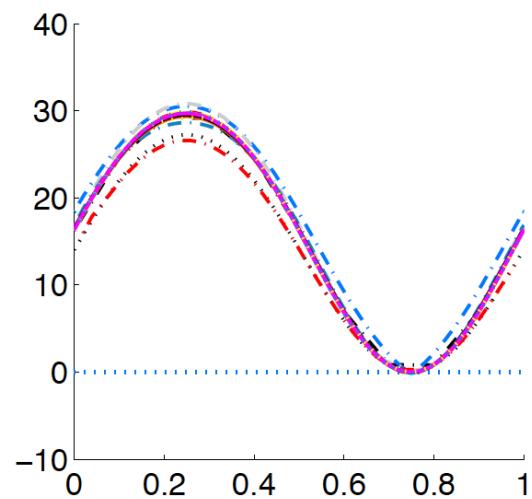
(a) V_x (m/yr) computed by ISSM (Pattyn) on
a cross line



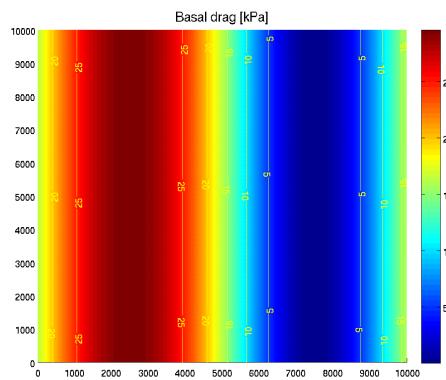
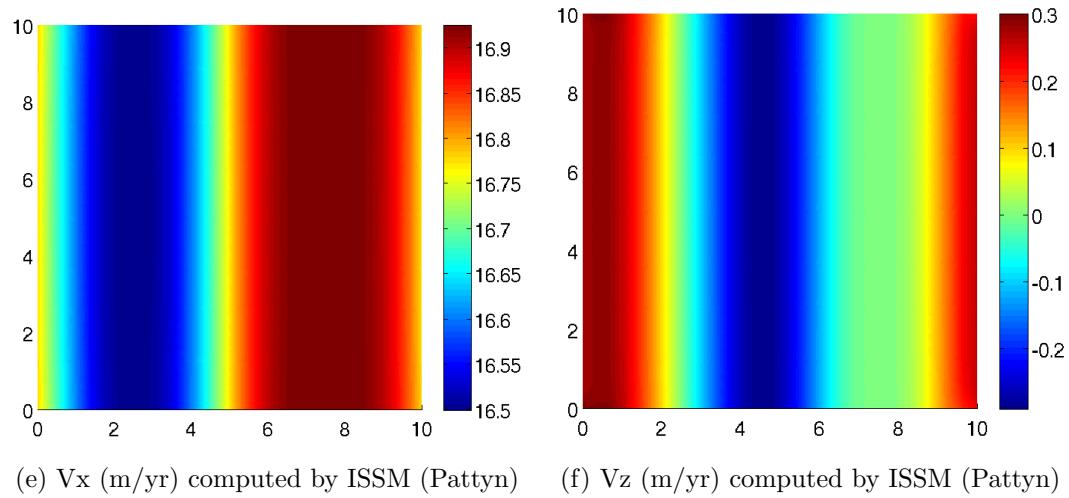
(b) V_x (m/yr) comparison



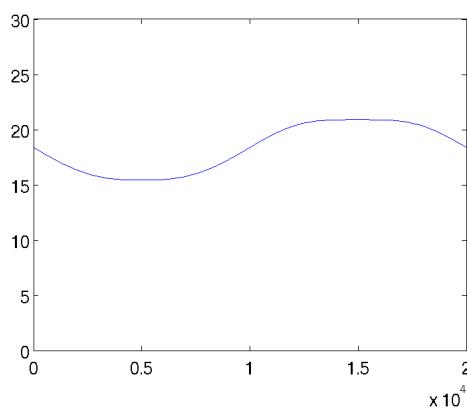
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on a cross line



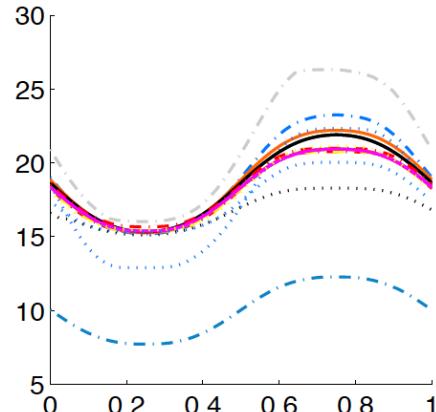
(d) T_{xz} (kPa) comparison



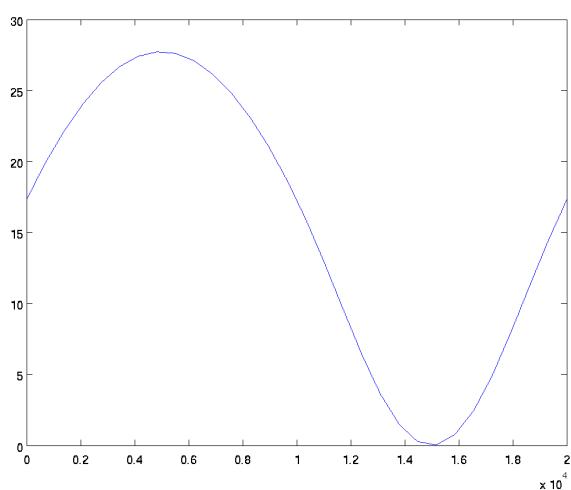
3.4.2.3 20km



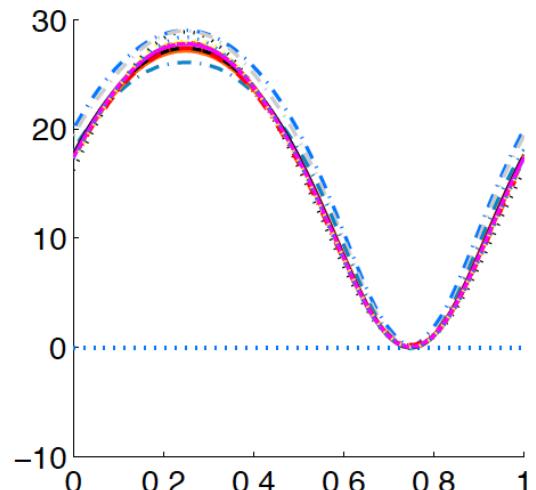
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



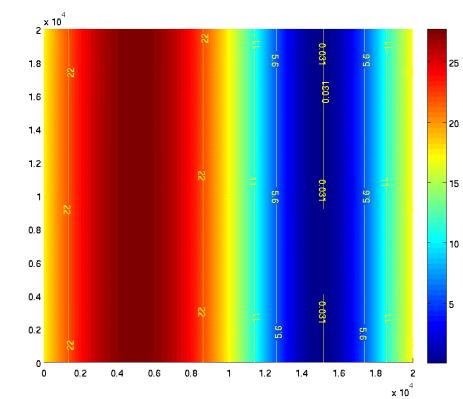
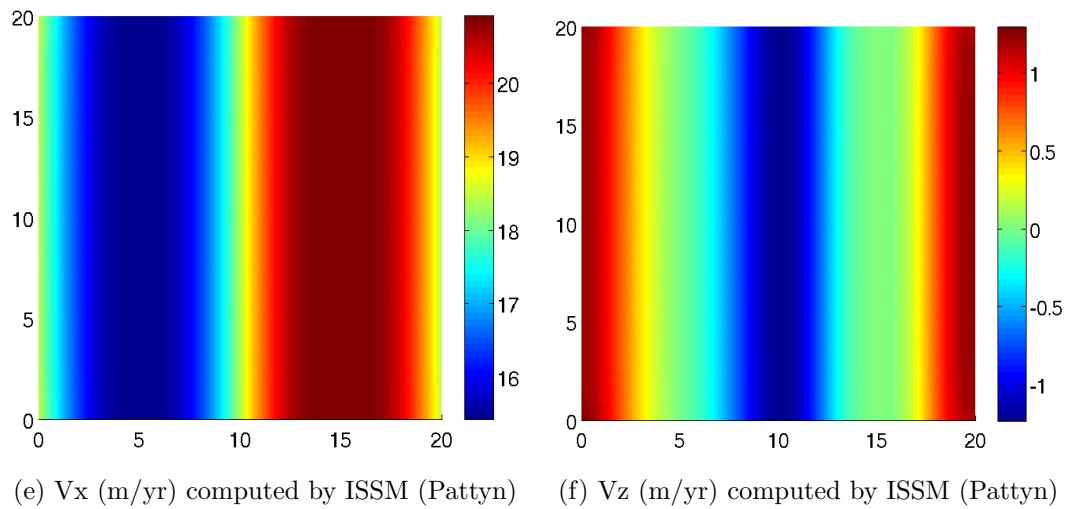
(b) V_x (m/yr) comparison



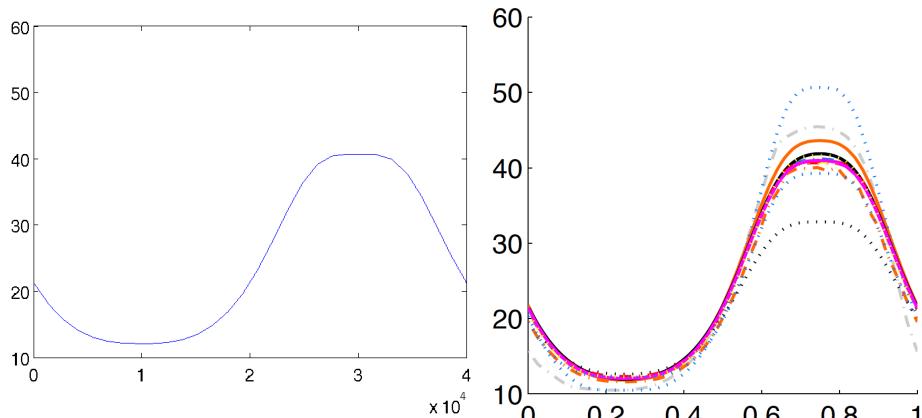
(c) T_{xz} (kPa) computed by ISSM (Pattyn)
on a cross line



(d) T_{xz} (kPa) comparison

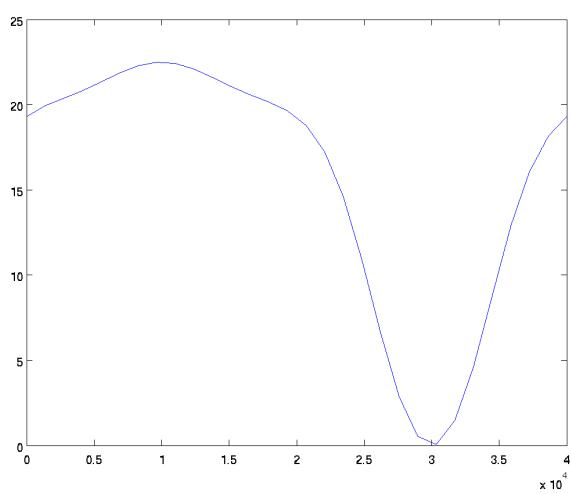


3.4.2.4 40km

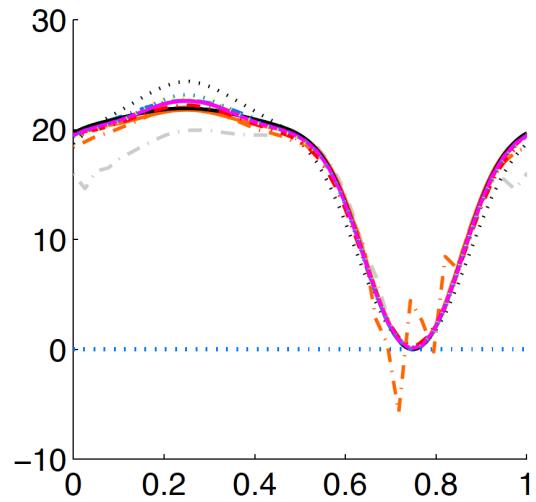


(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line

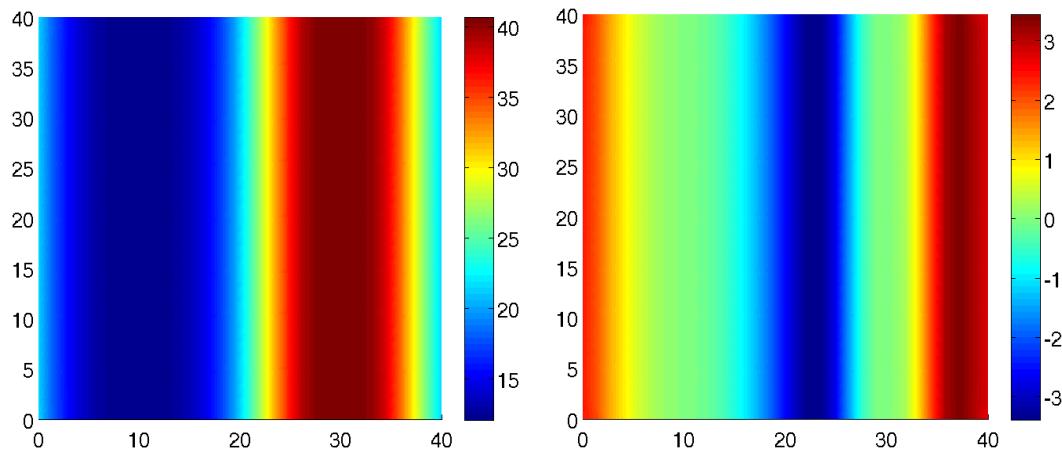
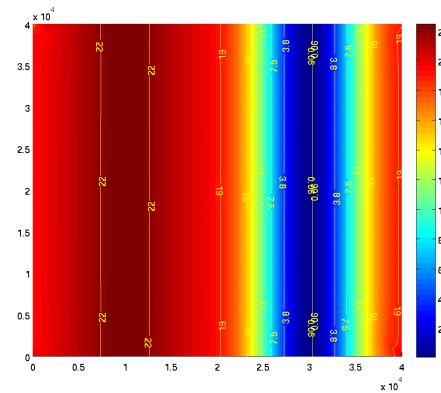
(b) V_x (m/yr) comparison



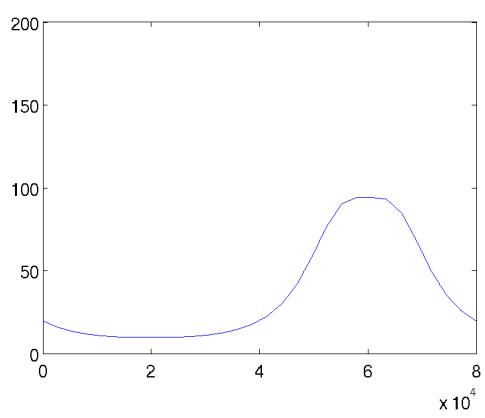
(c) T_{xz} (kPa) computed by ISSM (Pattyn) on a cross line



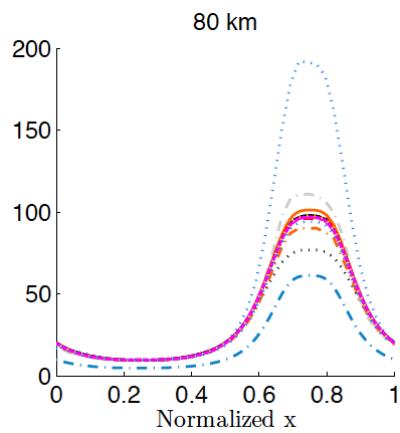
(d) T_{xz} (kPa) comparison

(e) V_x (m/yr) computed by ISSM (Pattyn)(f) V_z (m/yr) computed by ISSM (Pattyn)(g) T_{xz} (kPa) computed by ISSM (Pattyn)

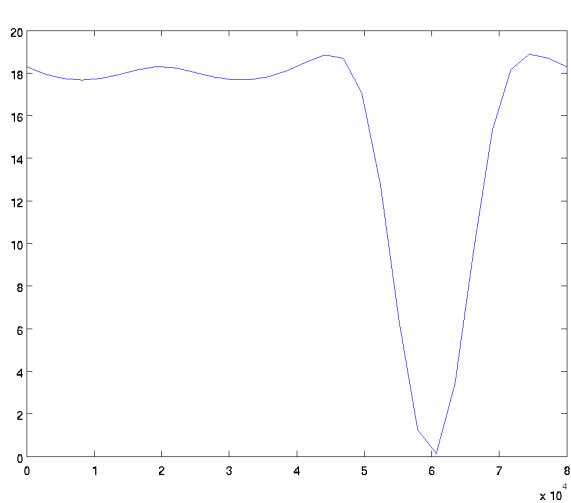
3.4.2.5 80km



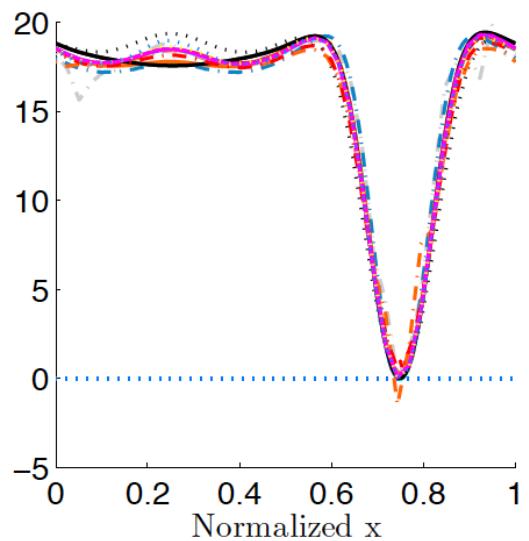
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line



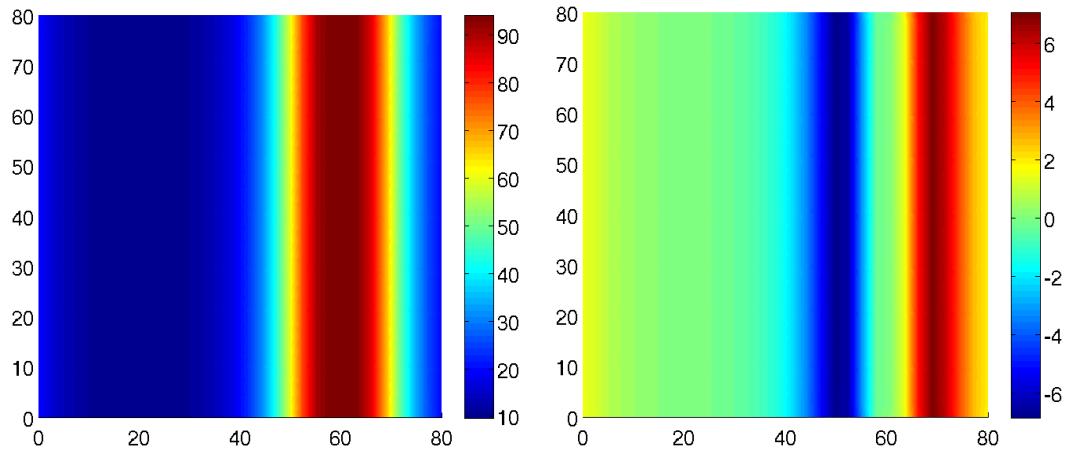
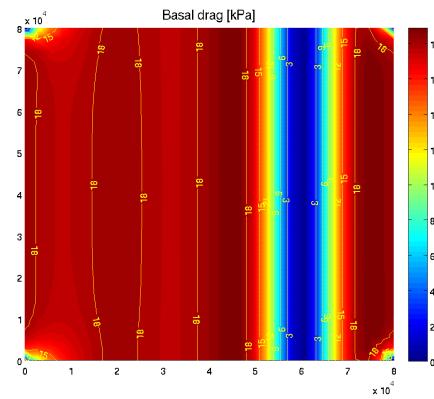
(b) V_x (m/yr) comparison



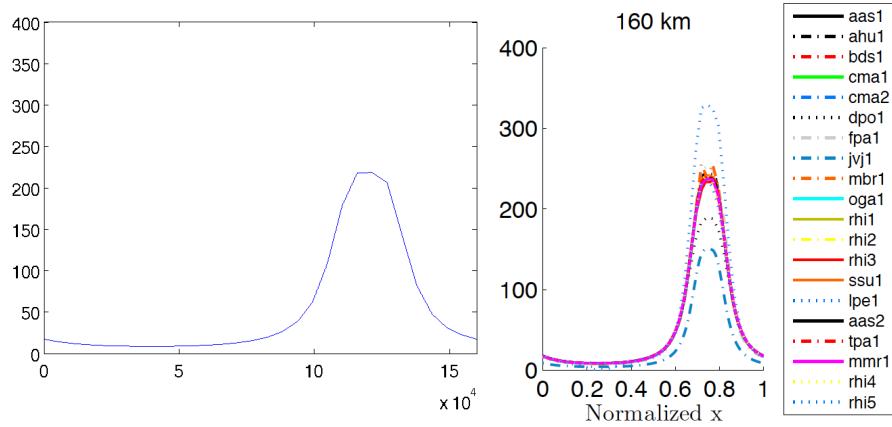
(c) T_{xz} (kPa) computed by ISSM (Pattyn)
on a cross line



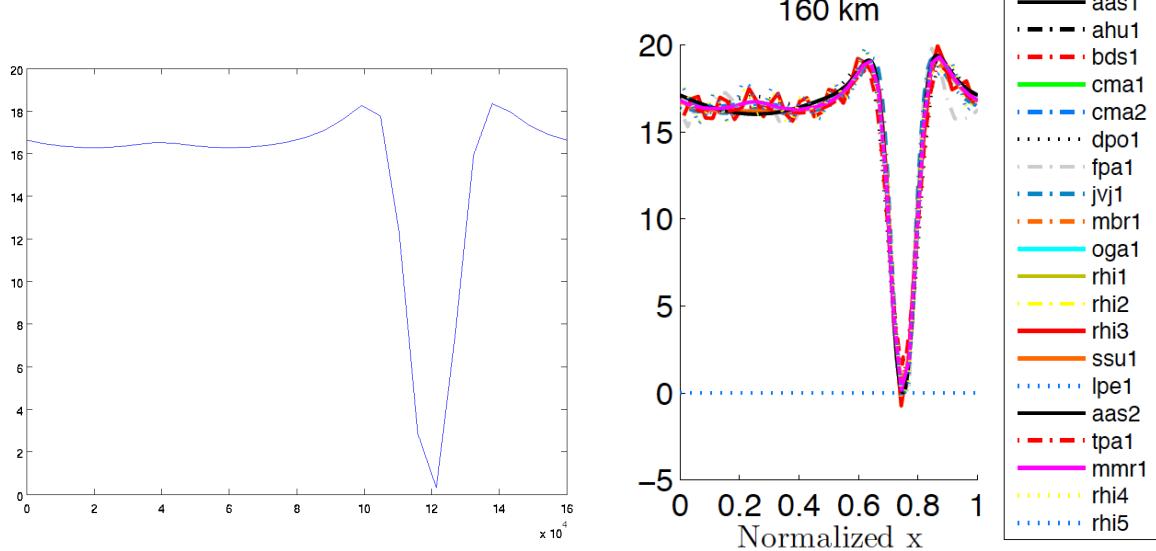
(d) T_{xz} (kPa) comparison

(e) V_x (m/yr) computed by ISSM (Pattyn) (f) V_z (m/yr) computed by ISSM (Pattyn)(g) T_{xz} (kPa) computed by ISSM (Pattyn)

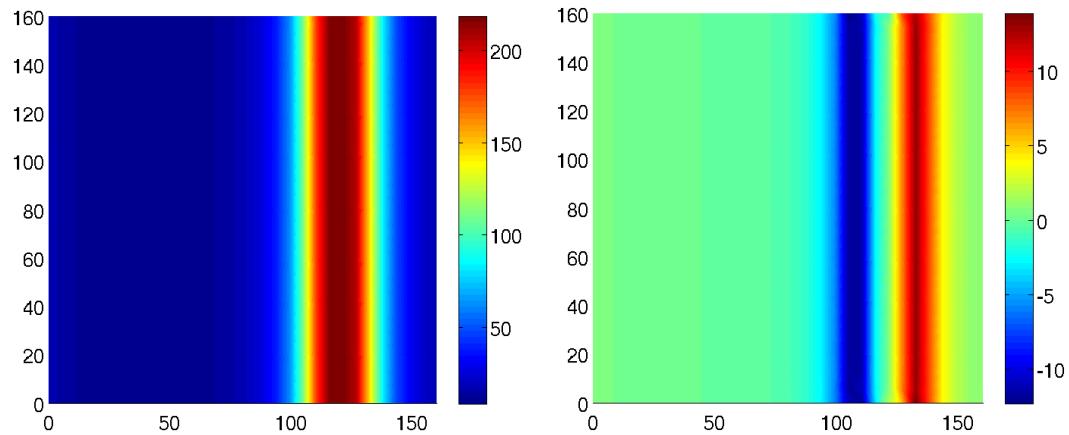
3.4.2.6 160km



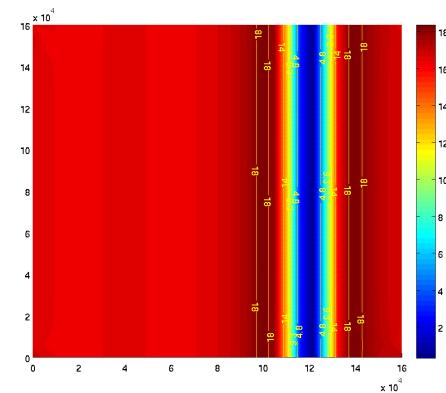
(a) V_x (m/yr) computed by ISSM (Pattyn)
on a cross line (b) V_x (m/yr) comparison



(c) T_{xz} (kPa) computed by ISSM (Pattyn) on a cross line (d) T_{xz} (kPa) comparison

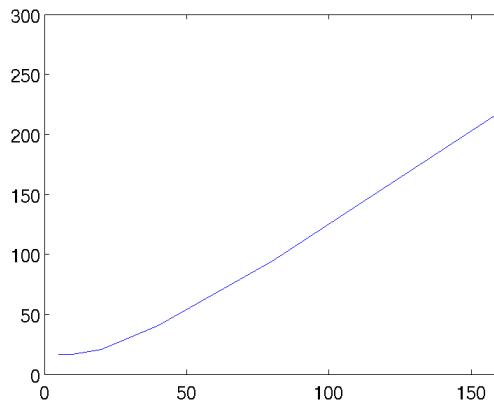


(e) V_x (m/yr) computed by ISSM (Pattyn) (f) V_z (m/yr) computed by ISSM (Pattyn)

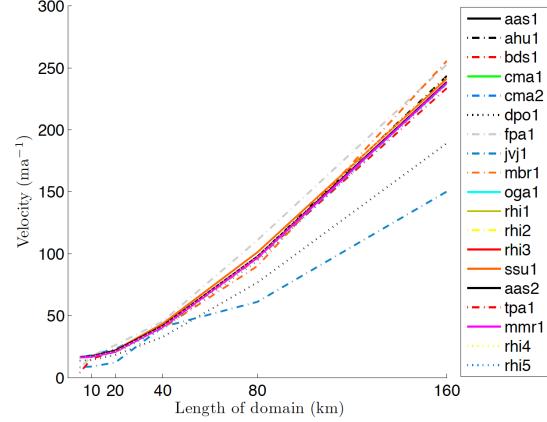


(g) T_{xz} (kPa) computed by ISSM (Pattyn)

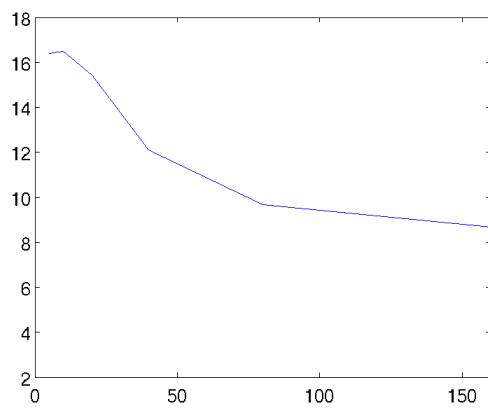
3.4.2.7 global (comparison)



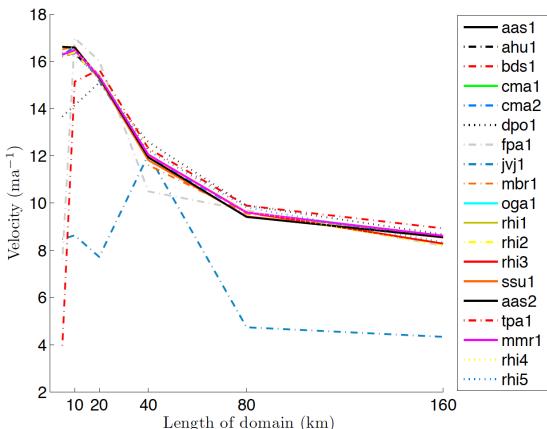
(h) Maximum Vx (m/yr) computed by ISSM (Pattyn)



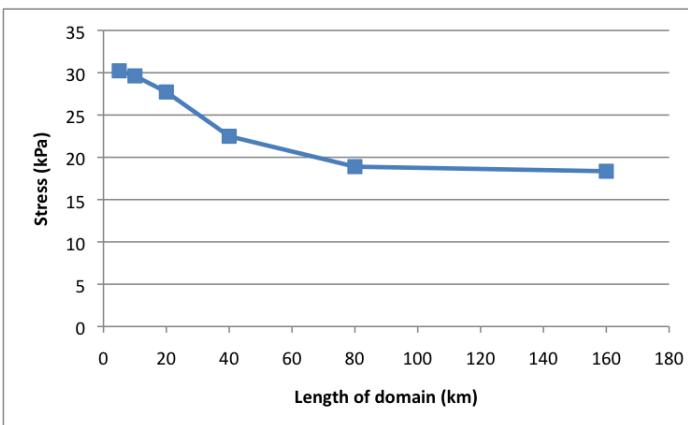
(i) Maximum Vx (m/yr) comparison



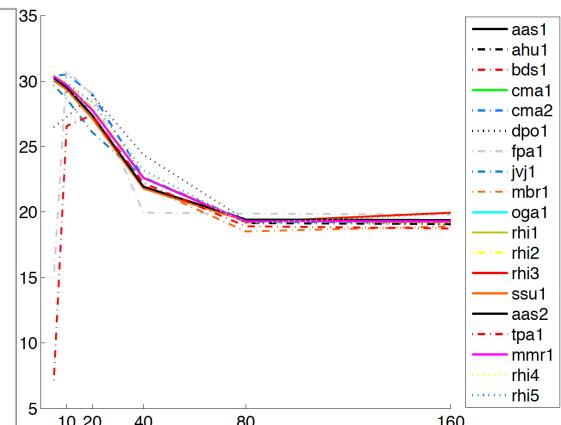
(j) Minimum Vx (m/yr) computed by ISSM (Pattyn)



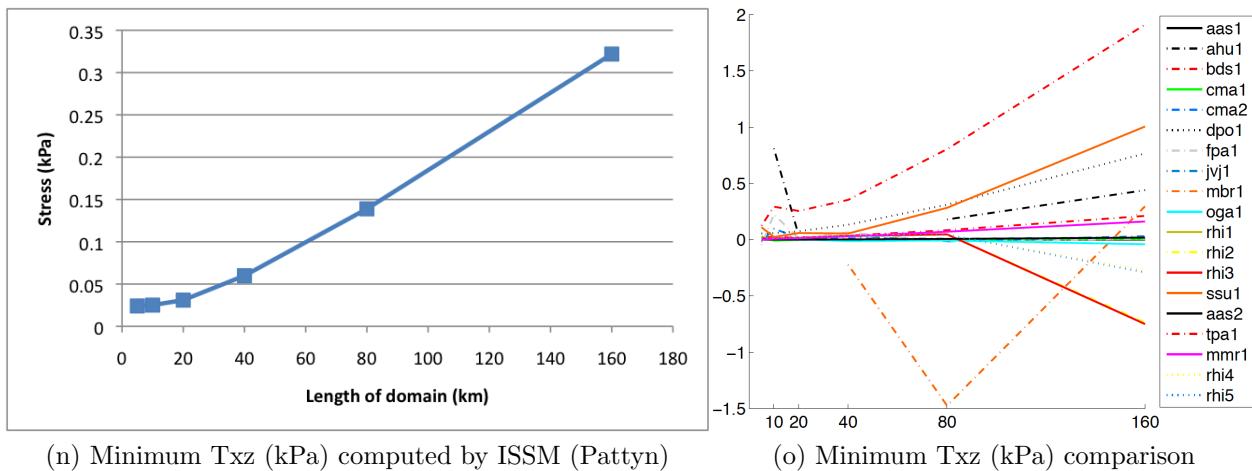
(k) Minimum Vx (m/yr) comparison



(l) Maximum Txz (kPa) computed by ISSM (Pattyn)



(m) Maximum Txz (kPa) comparison



3.5 Test E

3.5.1 Geometry

This experiment deals with the cross section of a real glacier: *Haut Glacier d'Arolla*. Two cases are studied:

- basal velocity = 0 on the bed
- sliding $\alpha^2(x, y) = 0$ for $0.44 \leq x \leq 0.5$

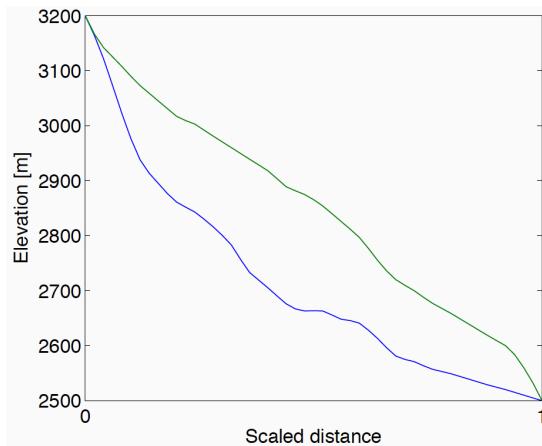
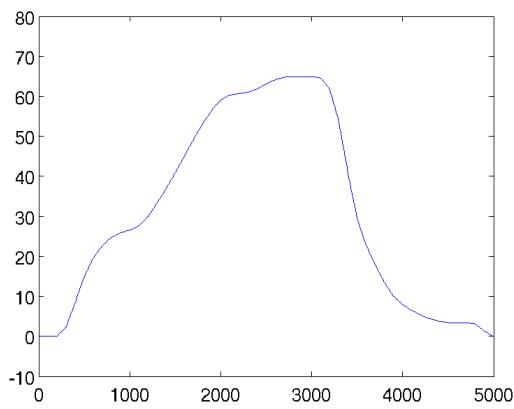


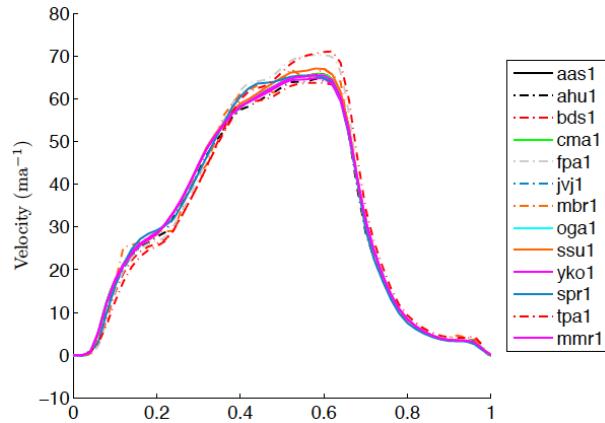
Figure 3.5: Test E geometry

3.5.2 Results

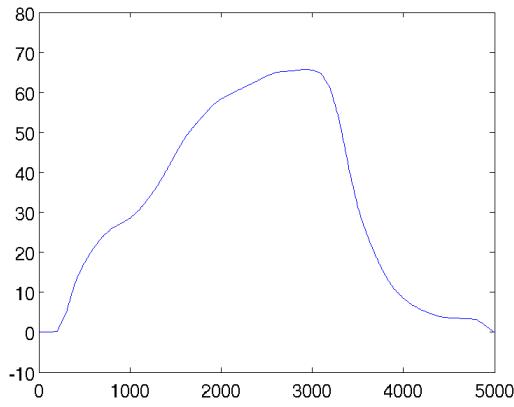
3.5.2.1 Frozen Bed



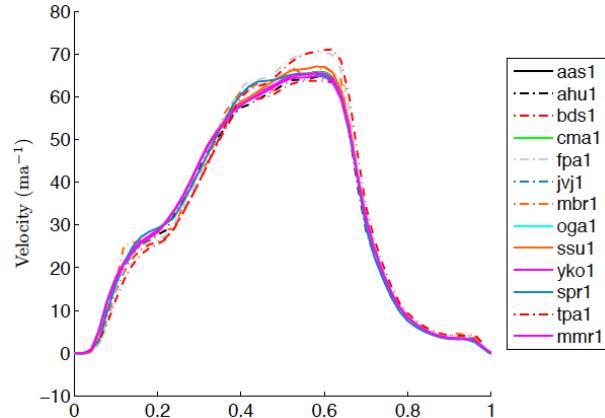
(a) V_x (m/yr) computed by ISSM (Pattyn) on a cross line



(b) V_x (m/yr) comparison

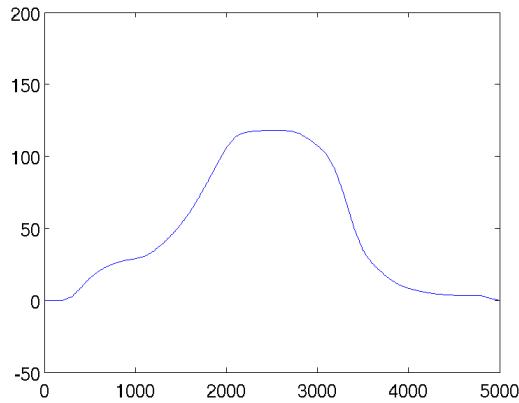


(c) V_x (m/yr) computed by ISSM (Stokes) on a cross line

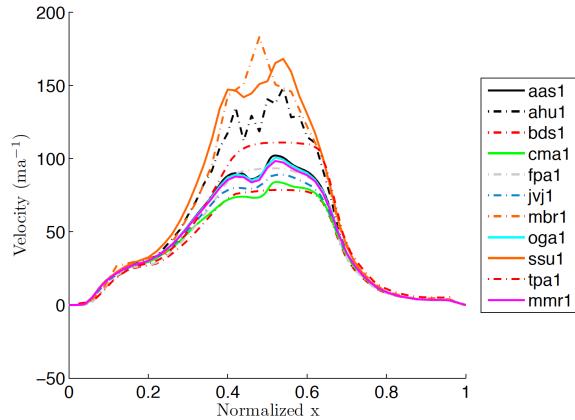


(d) V_x (m/yr) comparison

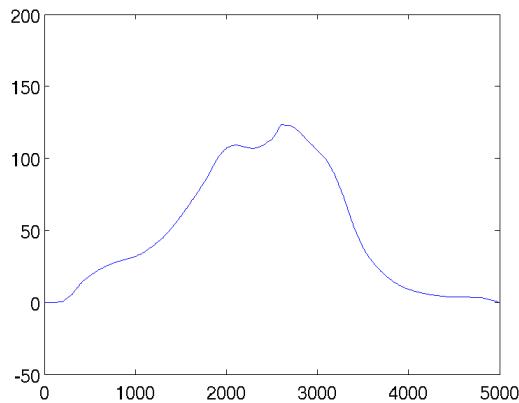
3.5.2.2 Sliding



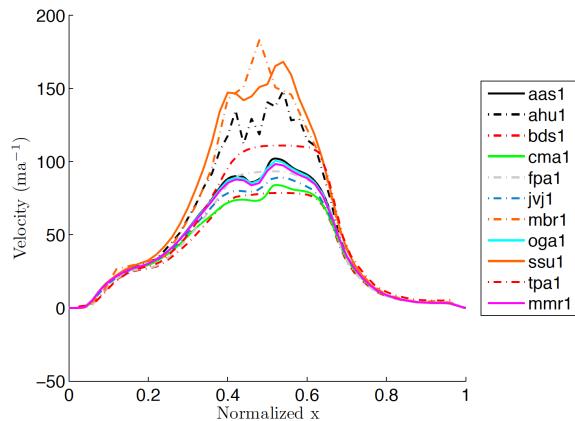
(a) V_x (m/yr) computed by ISSM (Pattyn) on a cross line



(b) V_x (m/yr) comparison



(c) V_x (m/yr) computed by ISSM (Stokes) on a cross line



(d) V_x (m/yr) comparison

3.6 Test F

3.6.1 Geometry

The goal of this experiment is to find the steady-state solution of an ice sheet sliding on a plane with an average slope of 3° with a gaussian bump in the middle, so this test allow to check the evolutive model in three dimension. Two cases are studied: one with a frozen bed and the other one with sliding at the bed, so the boundary conditions are:

- basal velocity = 0 on the bed
- sliding $\alpha^2(x, y) = \frac{B}{H}$

where H is the ice thickness (1000 m) and B the ice stiffness parameter.

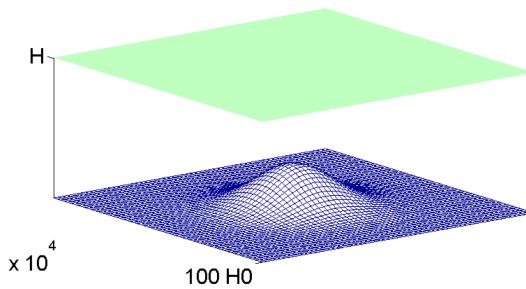
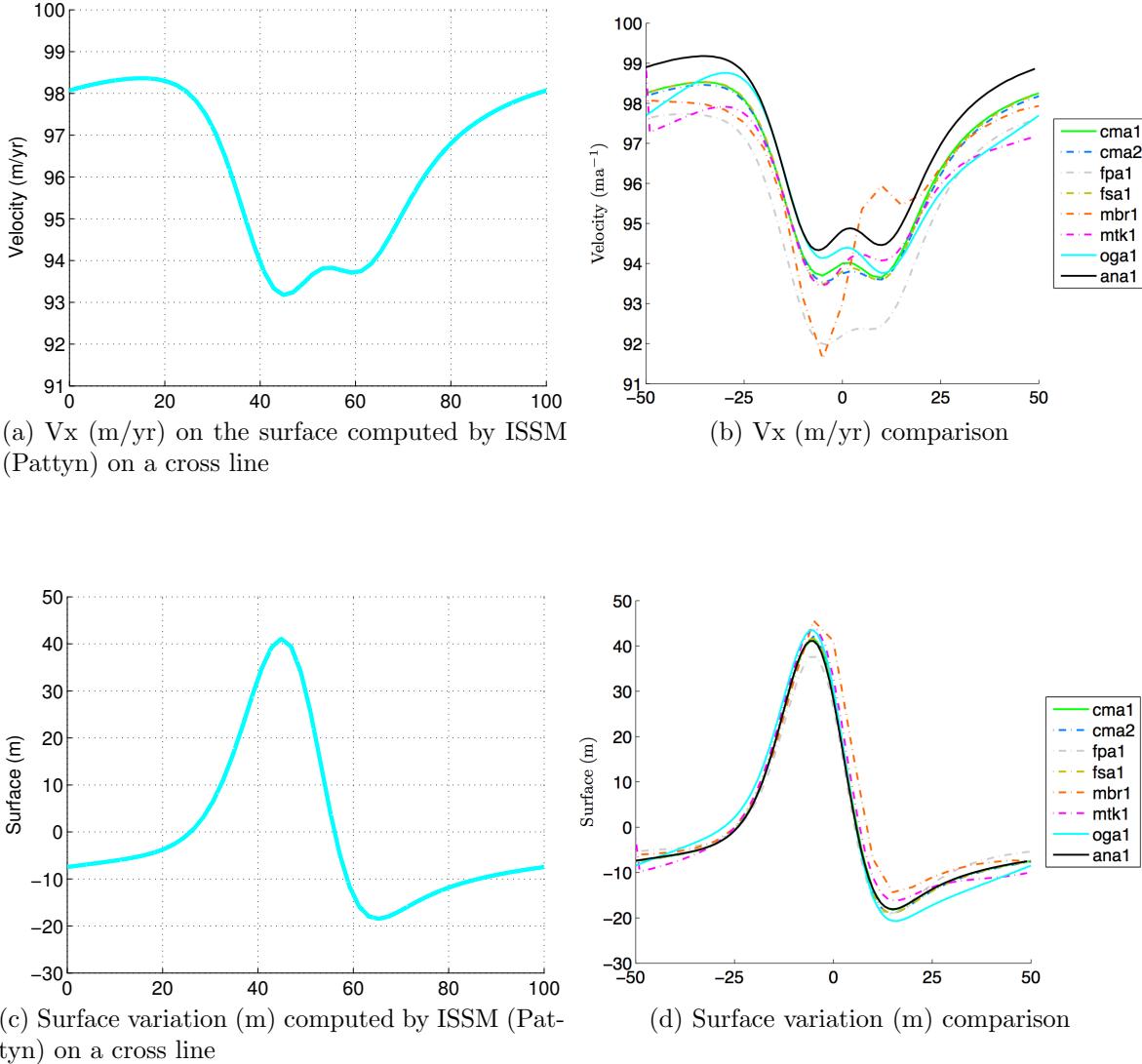


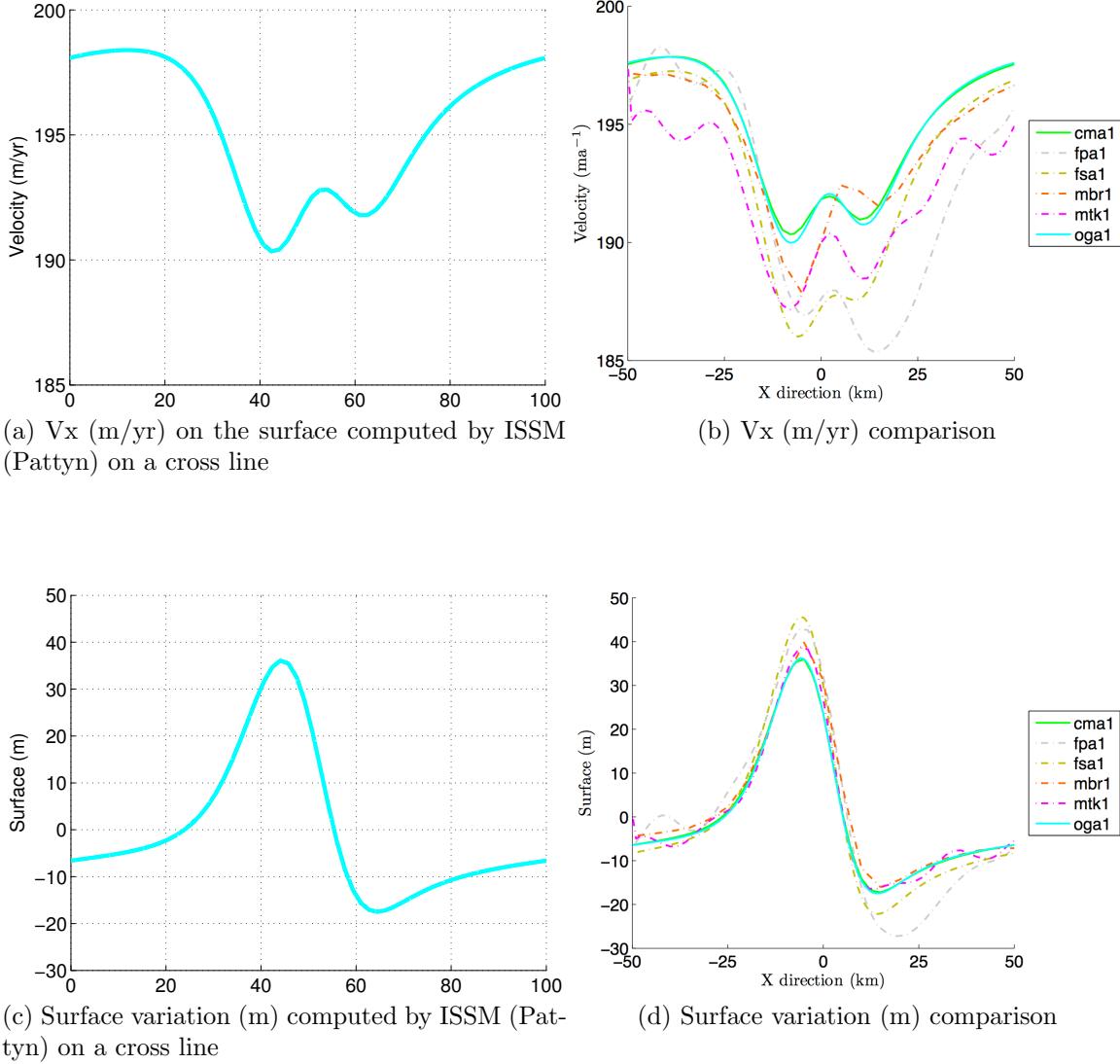
Figure 3.6: Test F geometry

3.6.2 Results for higher-order model

3.6.2.1 Frozen Bed

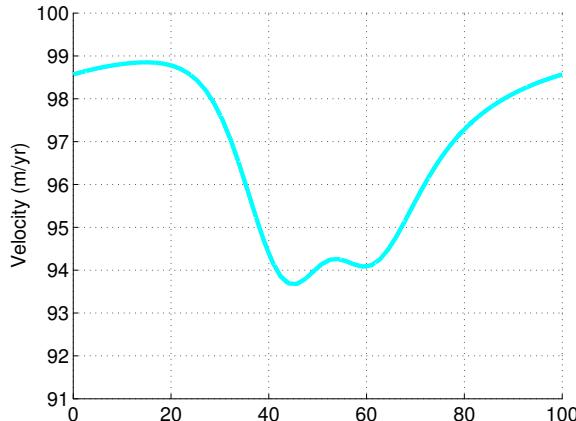


3.6.2.2 Sliding

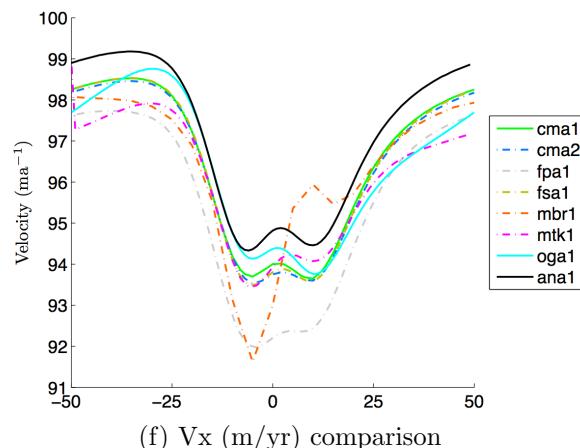


3.6.3 Results for full-Stokes model

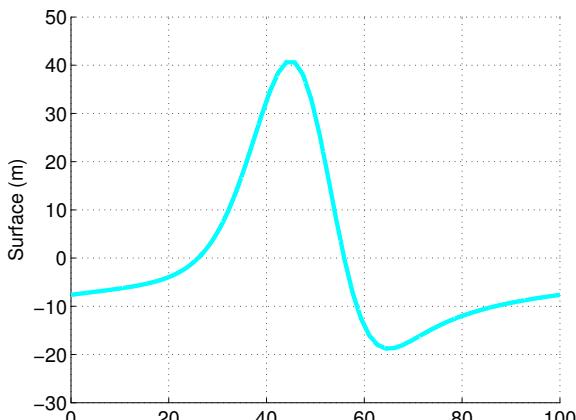
3.6.3.1 Frozen Bed



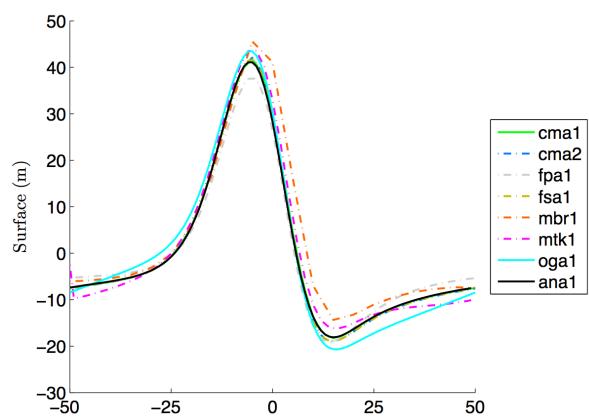
(e) V_x (m/yr) on the surface computed by ISSM (Stokes) on a cross line



(f) V_x (m/yr) comparison

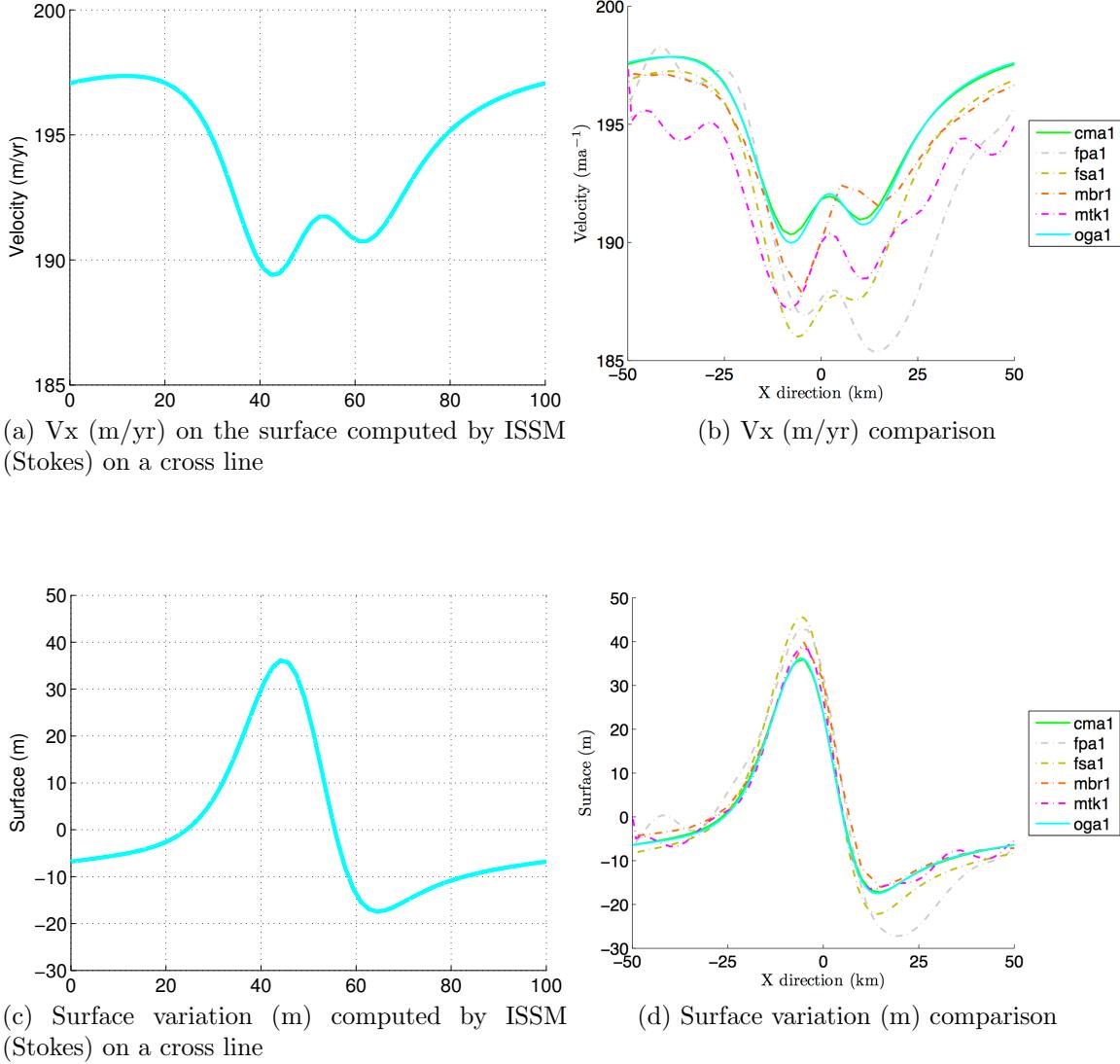


(g) Surface variation (m) computed by ISSM (Stokes) on a cross line



(h) Surface variation (m) comparison

3.6.3.2 Sliding



Chapter 4

Thermal

The tests described in this chapter validate the thermal model. The temperatures computed are compared with analytical results. The four tests in this chapter validate the advection, conduction, geothermal flux and the melting.

4.1 Vertical advection and conduction

4.1.1 Description

In this test, the thermal equation is reduced to vertical advection and conduction only: there is no geothermal flux and no horizontal velocity. The ice thickness, surface and bed are the same everywhere on the domain. On top of that we impose boundary conditions that do not vary with x and y so the solution depends only with z . The thermal equation is reduced to:

$$\frac{\partial^2 T}{\partial z^2} - w \frac{\rho_i c_p}{k} \frac{\partial T}{\partial z} = 0 \quad (4.1)$$

We impose Dirichlet conditions on the upper and lower surfaces, the temperature on the upper surface is set to $T = 0K$ and to $T = 10K$ on the lower surface. The analytical solution for this problem is:

$$T(z) = 10 \frac{e^{(\alpha z)} - e^{(\alpha b)}}{e^{(\alpha s)} - e^{(\alpha b)}} \quad (4.2)$$

with $\alpha = \frac{w \rho_i c_p}{k}$, where s is the ice surface, b is the ice base, ρ_i the ice density, c_p the heat capacity, k the thermal conductivity and w the vertical velocity.

4.1.2 Results

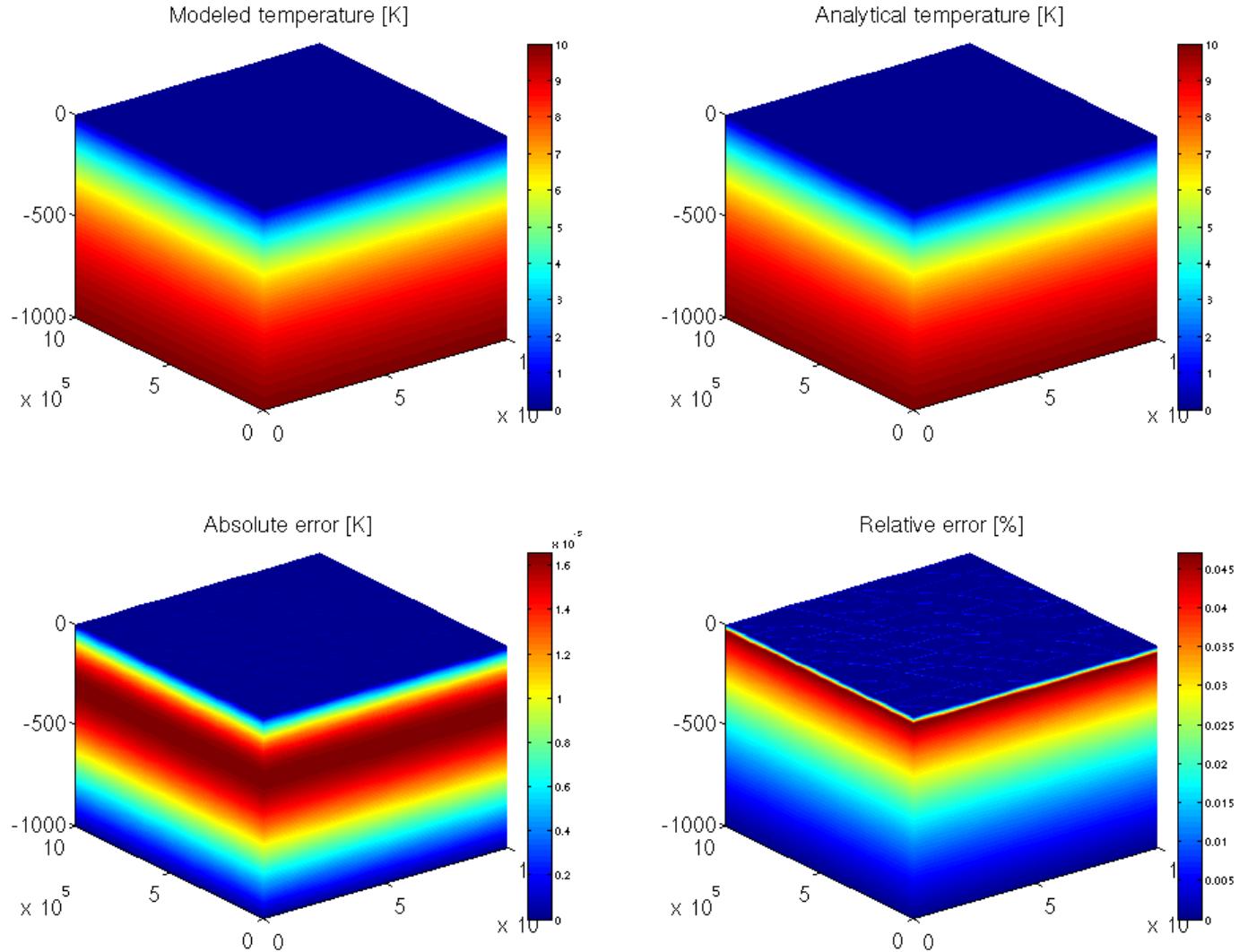


Figure 4.1: Comparison between modeled and analytical temperatures

4.2 Simple conduction

4.2.1 Description

In this test, the thermal equation is reduced to vertical conduction. The velocity (all three components) are null and there is no geothermal flux, so the thermal equation is reduced to conduction only. On top of that we put boundary conditions that do not vary with x and y , so the solution does not depend on these coordinates and therefore varies only with the vertical axes z . The ice sheet is square and the thickness, bed and surface are the same everywhere. The thermal equation is:

$$\frac{\partial^2 T}{\partial z^2} = 0 \quad (4.3)$$

We impose Dirichlet conditions at the upper and lower surface, the temperature at the upper surface is set to $T = 0K$ and to $T = 10K$ on the lower layer. The analytical solution for this problem is:

$$T = 10 \frac{s - z}{H} \quad (4.4)$$

where H is the ice thickness, b is the ice base and s is the ice surface.

4.2.2 Results

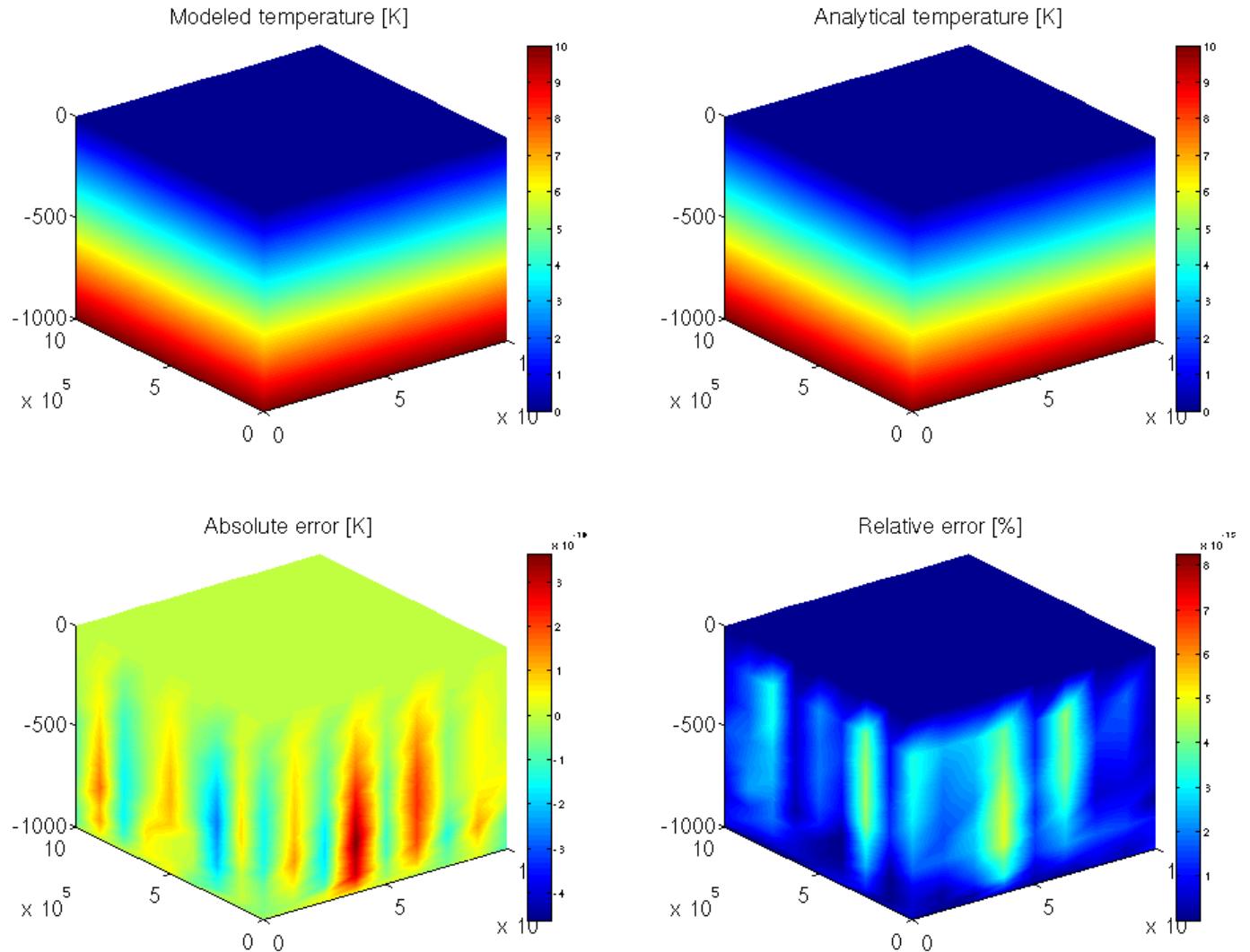


Figure 4.2: Comparison between modeled and analytical temperatures

4.3 Simple geothermal flux

4.3.1 Description

In this test we want to validate the boundary condition on the geothermal flux. We use a zero velocity so there is no advection. As for the previous tests, we impose boundary conditions that do not depend on x or y so the solution varies only with z . The heat equation is therefore reduced to:

$$\frac{\partial^2 T}{\partial z^2} = 0 \quad (4.5)$$

We impose a Dirichlet condition on the upper surface: $T = 0K$ and a Neumann boundary condition on the lower surface: a geothermal flux G such that $-k * dT/dz(bed) = G$, with k the thermal conductivity. So the analytical solution of this equation with these boundary condition is:

$$T(z) = \frac{G}{K} (s - z) \quad (4.6)$$

with s the ice surface, G the geothermal flux and k the thermal conductivity.

4.3.2 Results

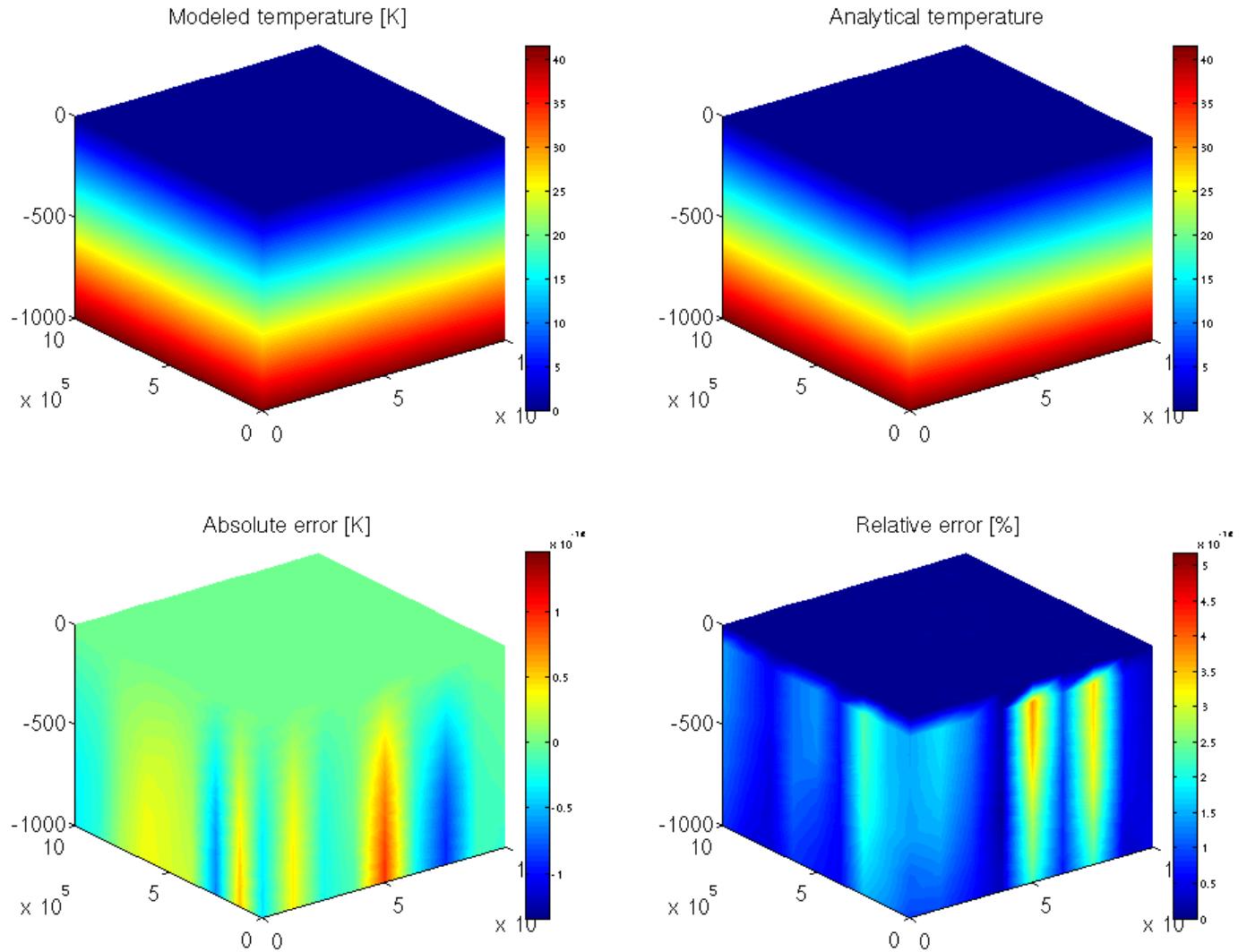


Figure 4.3: Comparison between modeled and analytical temperatures

4.4 Melting

4.4.1 Description

In this test, we want to check the computed melting rate. To do so we use a model with a temperature at the melting point ($273, 15K$) everywhere and we then add a geothermal flux. As the temperature is at the melting point everywhere, the extra heat from the geothermal flux can only melt ice. The energy used to melt the ice is the energy brought by the geothermal flux so:

$$Mb = \frac{G}{L\rho_i} \quad (4.7)$$

where Mb is the amount of ice melted in a year, G is the geothermal flux, L is ice latent heat and ρ_i is the ice density.

4.4.2 Results

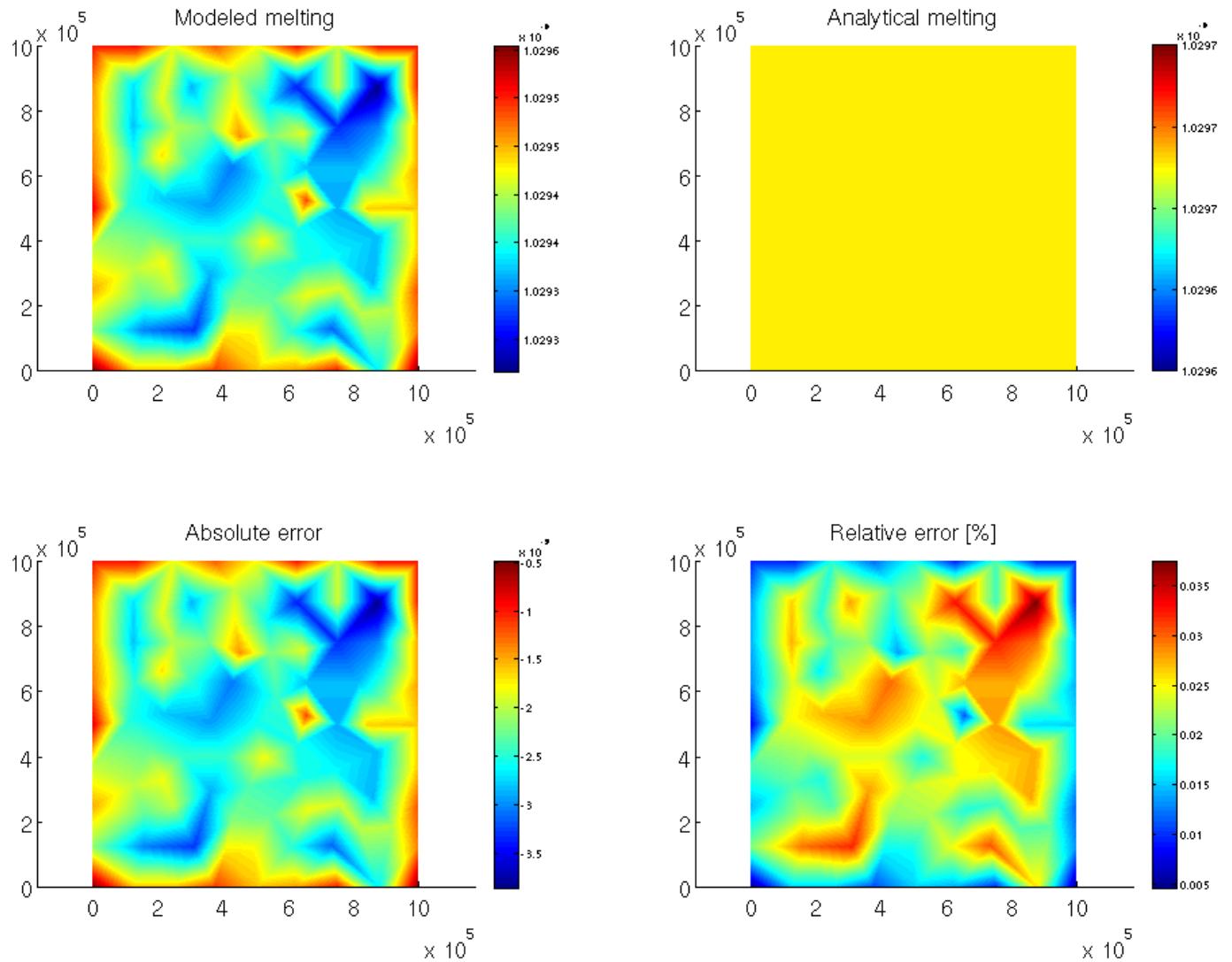


Figure 4.4: Comparison between modeled and analytical temperatures

Chapter 5

Anisotropic mesh adaptation

5.1 Test A

We are trying to adapt an anisotropic mesh according to the following solution:

$$f(x, y) = \exp\left(-\left(\frac{r - 0.75}{10^{-3}}\right)^2\right) + 0.5r^2 \quad \text{with} \quad r = \sqrt{(x + 0.1)^2 + (y + 0.1)^2} \quad (5.1)$$

An *isotropic* mesh for a given interpolation error of 0.005 gives the following mesh of 22,500 triangles:

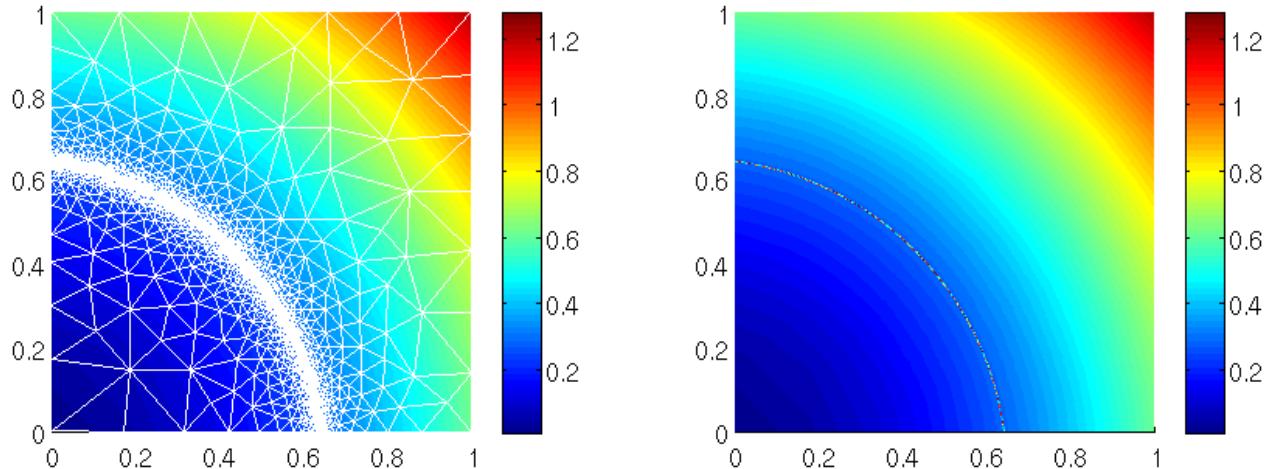
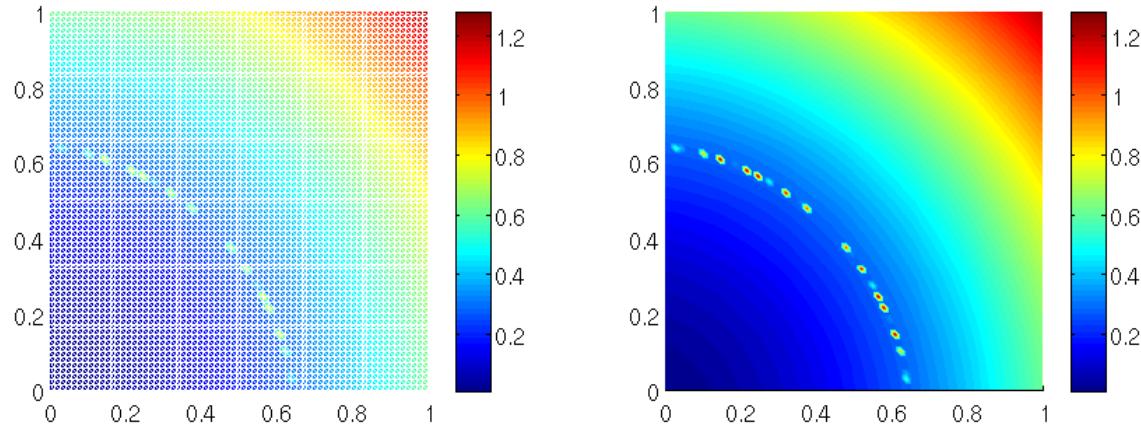
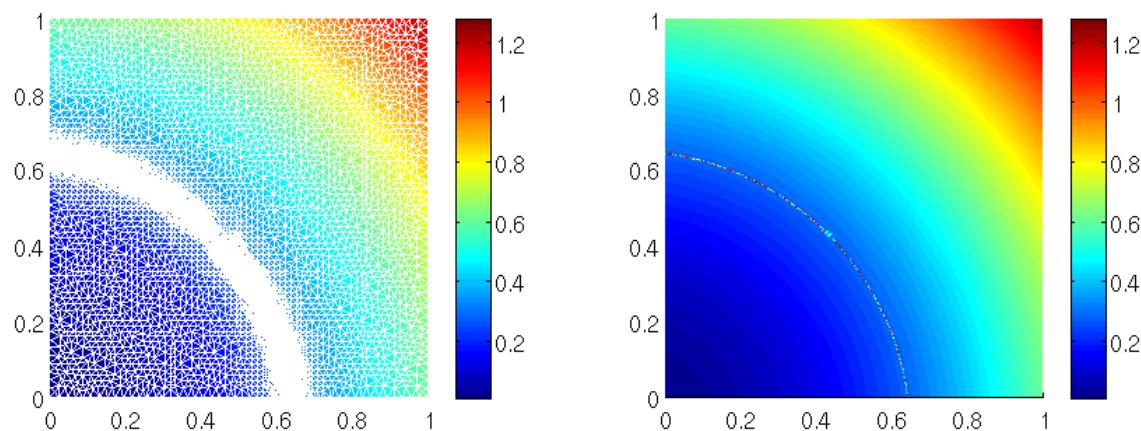


Figure 5.1: Adapted isotropic mesh of 22,500 elements

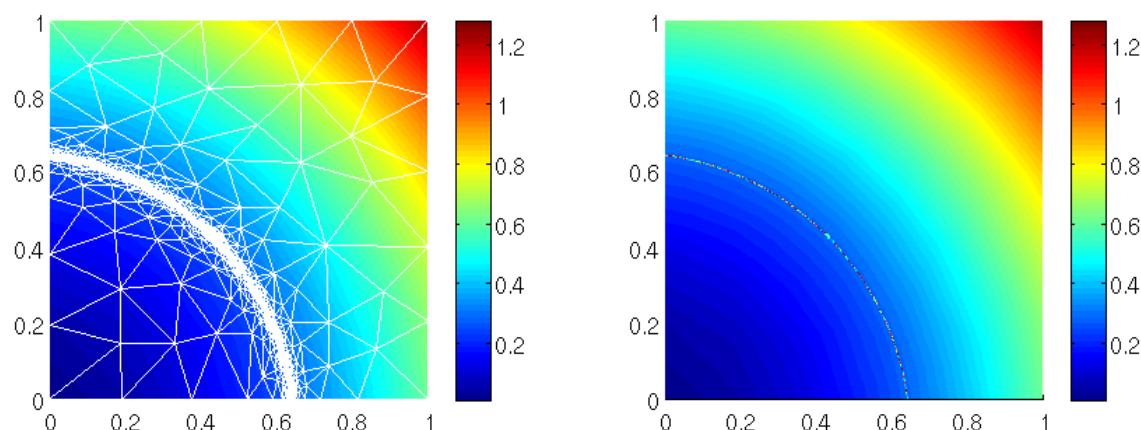
5.1.1 Bamg



(a) Initial Mesh

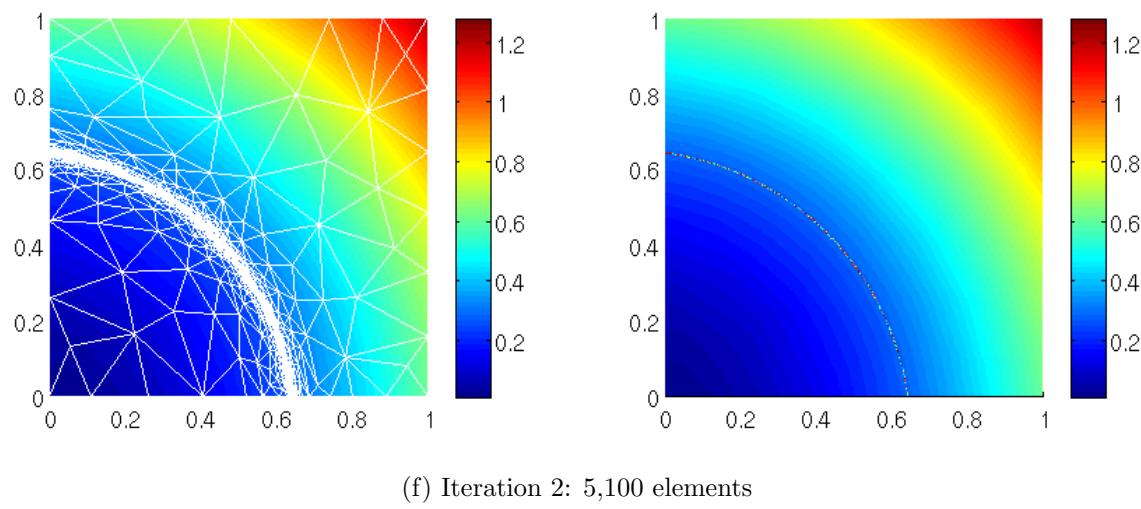
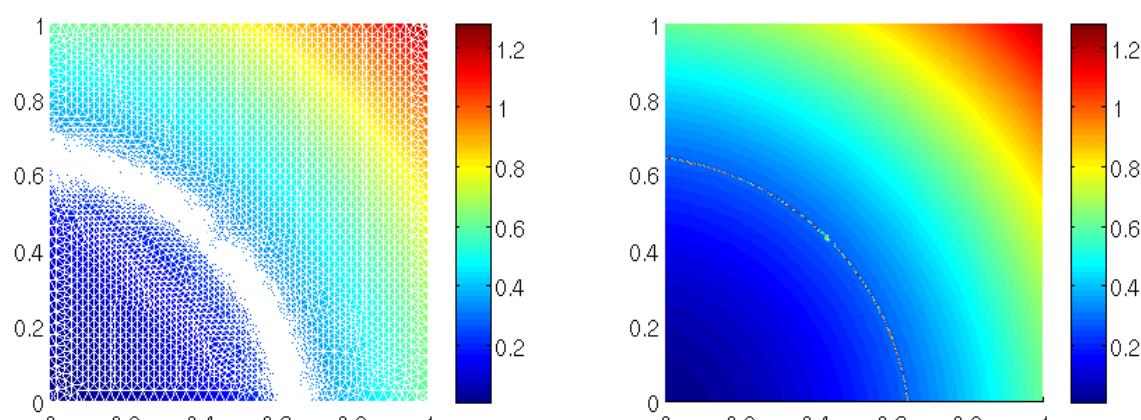
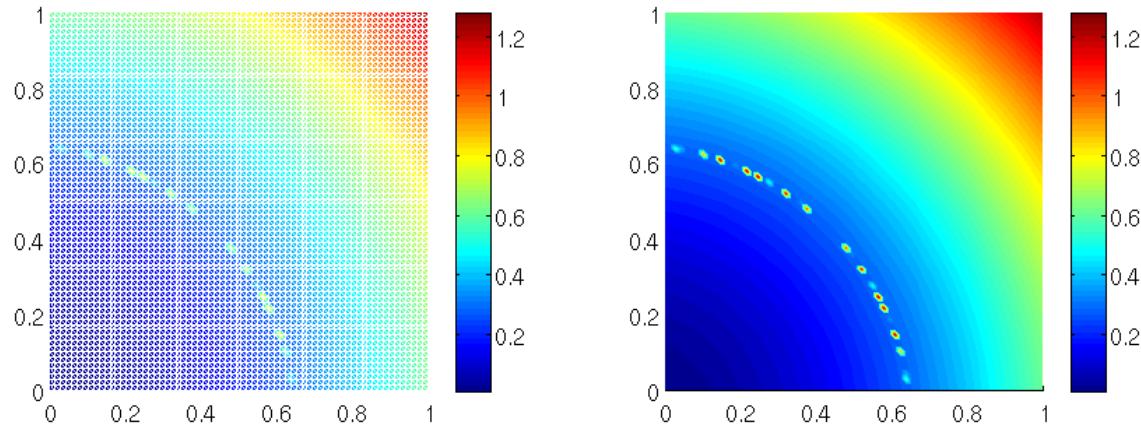


(b) Iteration 1: 37,000 elements



(c) Iteration 2: 7,600 elements

5.1.2 Yams



5.2 Test B

We are trying to adapt an anisotropic mesh according to the following solution:

$$\begin{aligned}
 f(x, y) = & \tanh(30(u^2 + v^2 - \varepsilon)) \\
 & + \tanh(30((u - 0.75)^2 + (v - 0.75)^2 - \varepsilon)) + \tanh(30((u - 0.75)^2 + (v - 0.75)^2 + \varepsilon)) \\
 & + \tanh(30((u - 0.75)^2 - (v - 0.75)^2 + \varepsilon)) + \tanh(30((u - 0.75)^2 - (v - 0.75)^2 - \varepsilon))
 \end{aligned}$$

with $\varepsilon = 0.25$ and $u = 4x - 2, v = 4y - 2$

(5.2)

An *isotropic* mesh for a given interpolation error of 0.005 gives the following mesh of 22,500 triangles:

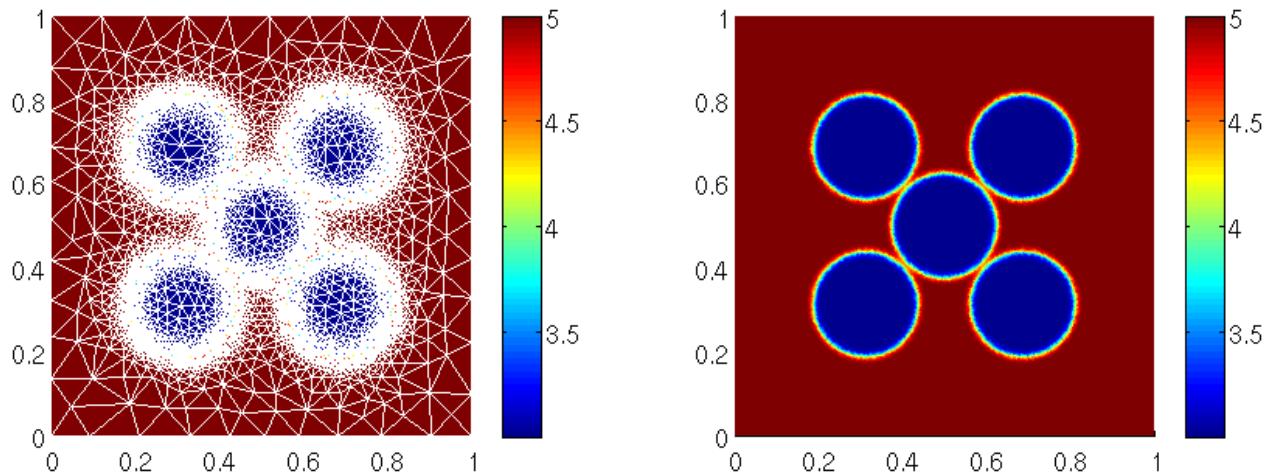
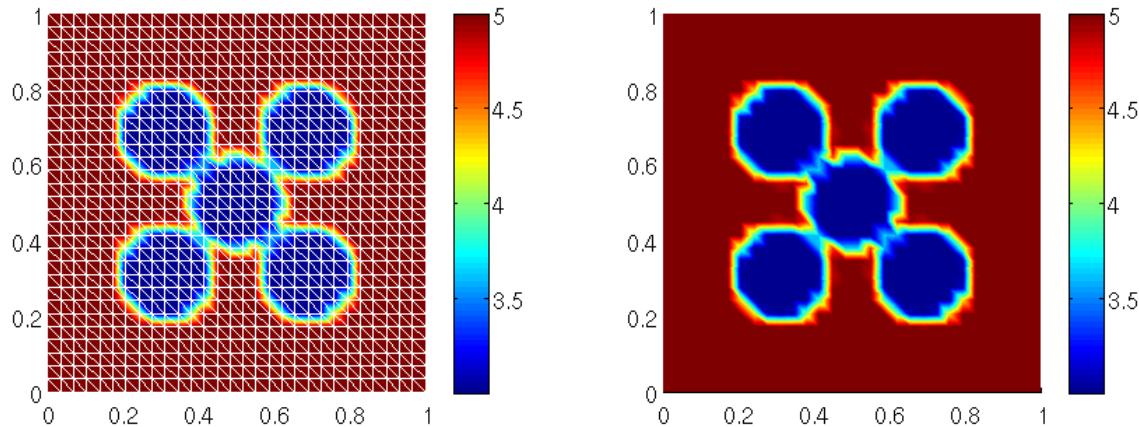
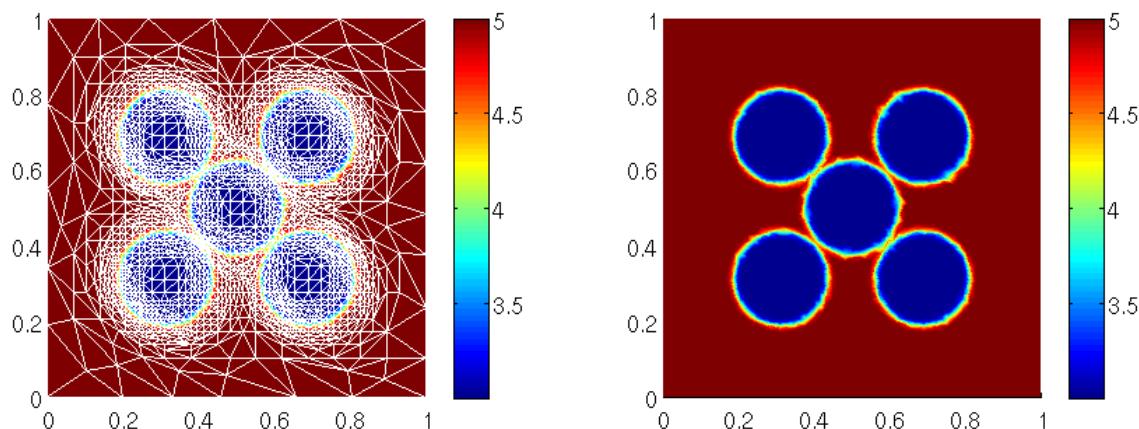


Figure 5.2: Adapted isotropic mesh of 22,500 elements

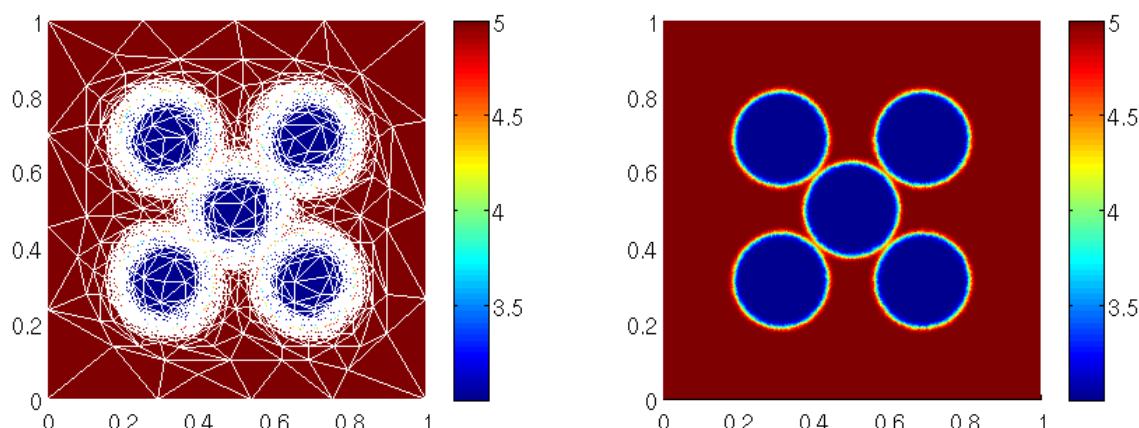
5.2.1 Bamg



(a) Initial Mesh

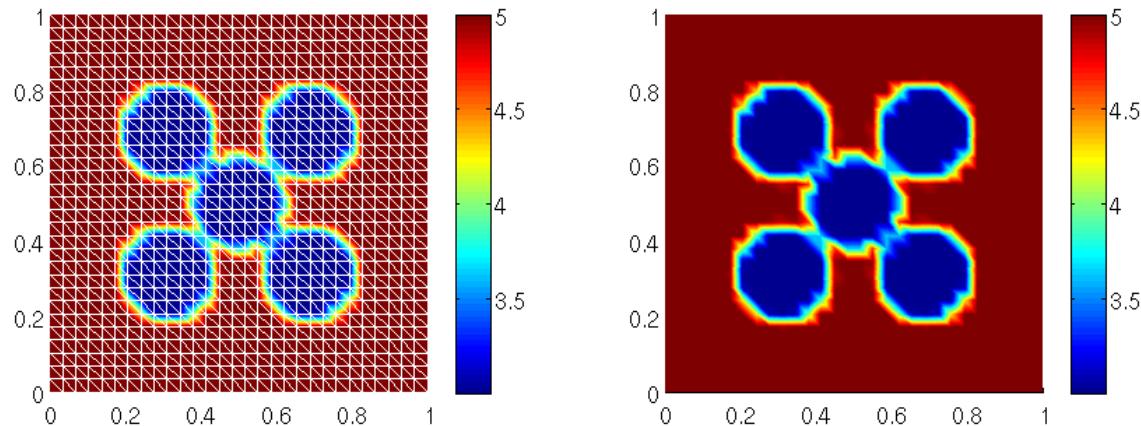


(b) Iteration 1: 4,500 elements

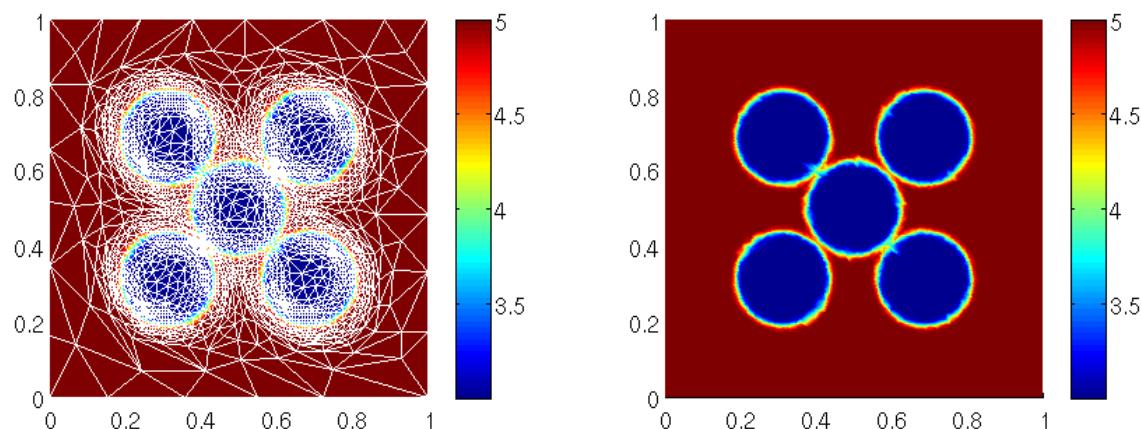


(c) Iteration 2: 10,000 elements

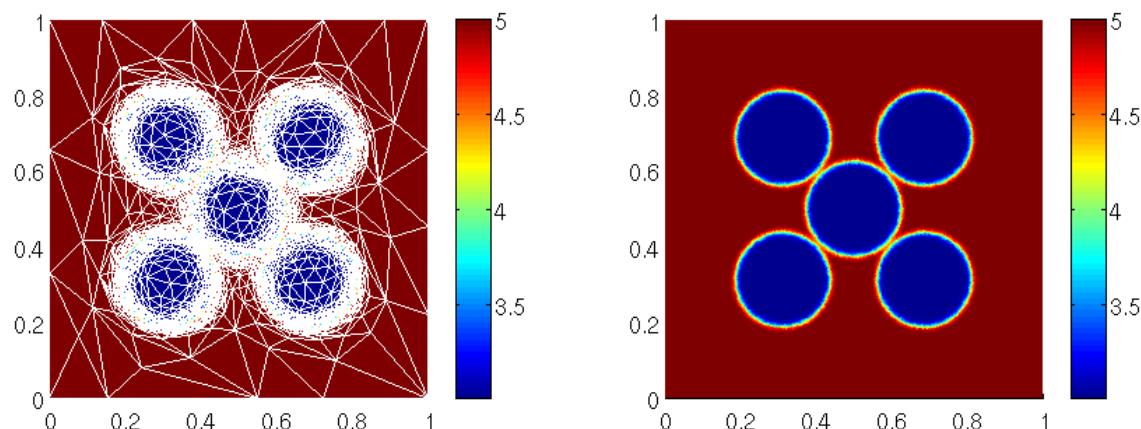
5.2.2 Yams



(d) Initial Mesh



(e) Iteration 1: 4,500 elements



(f) Iteration 2: 10,000 elements

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